

# **Logistic Regression**

**DS 4400 | Machine Learning and Data Mining I**

**Zohair Shafi**

**Spring 2026**

**Monday | February 9, 2026**

# Updates

- Homework 1 Discussion
- Homework 3 Out - Due March 6th

# Updates

- 17th Feb - Tuesday - Wanrou
- 1:30 PM - 3:00 PM | Location: Richards Hall 243
  - Linear algebra
    - Vectors
    - Matrices
    - Vector and Matrix operations
  - Probabilities
    - Bayes' rule and conditional probability
    - Distributions
    - CDFs and PDFs
- 18th Feb - Wednesday - Zaiba
- 1:00 PM - 2:30 PM | Location: EL 311
  - Derivatives
  - Gradients
  - Derivatives of some common functions
  - Chain Rule, Product Rule, Quotient Rule

# Today's Outline

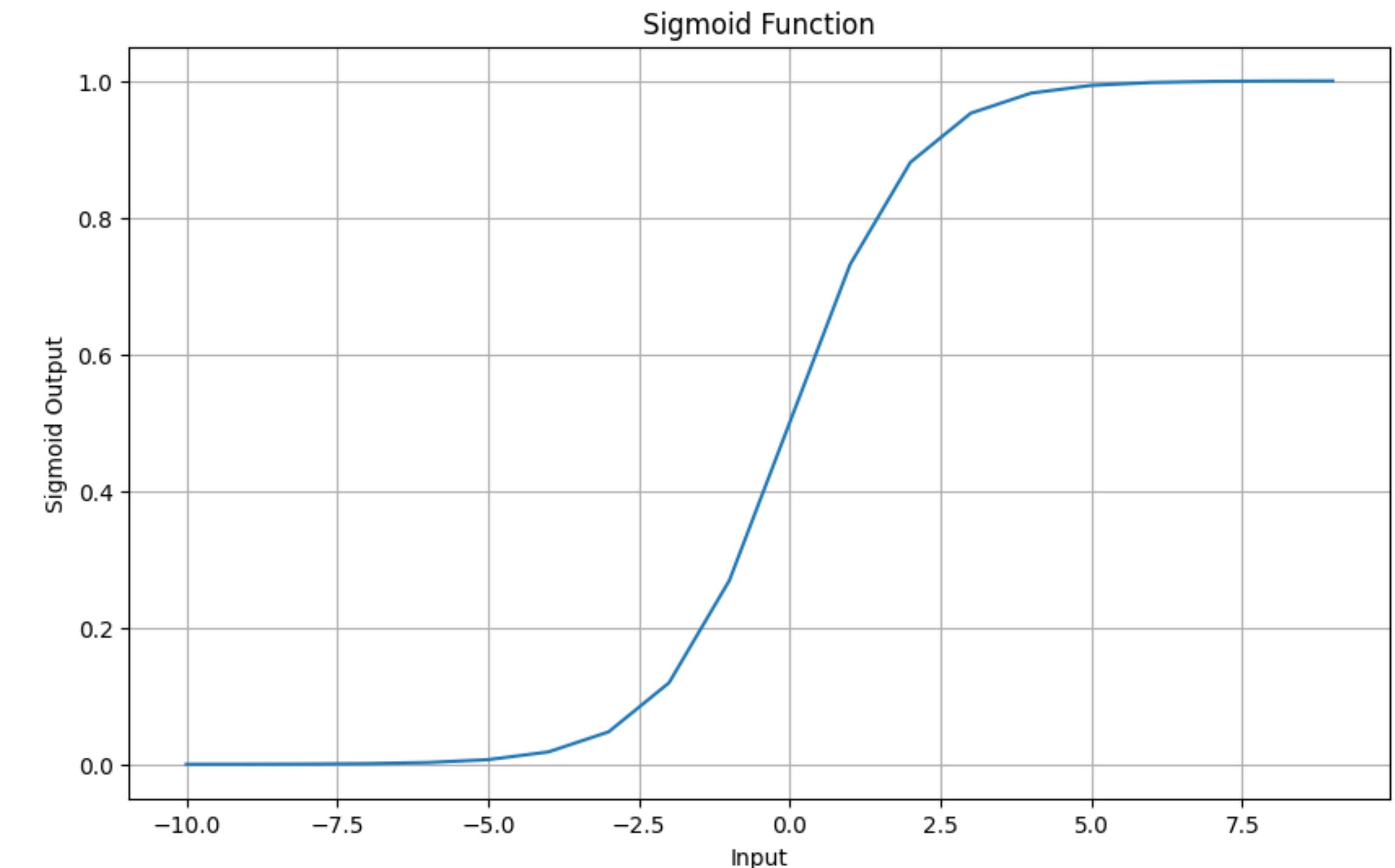
- Logistic Regression

# Logistic Regression

# Logistic Regression

- Despite its name, logistic regression is a **classification** algorithm.
- It models the probability of class membership using a logistic (sigmoid) function.

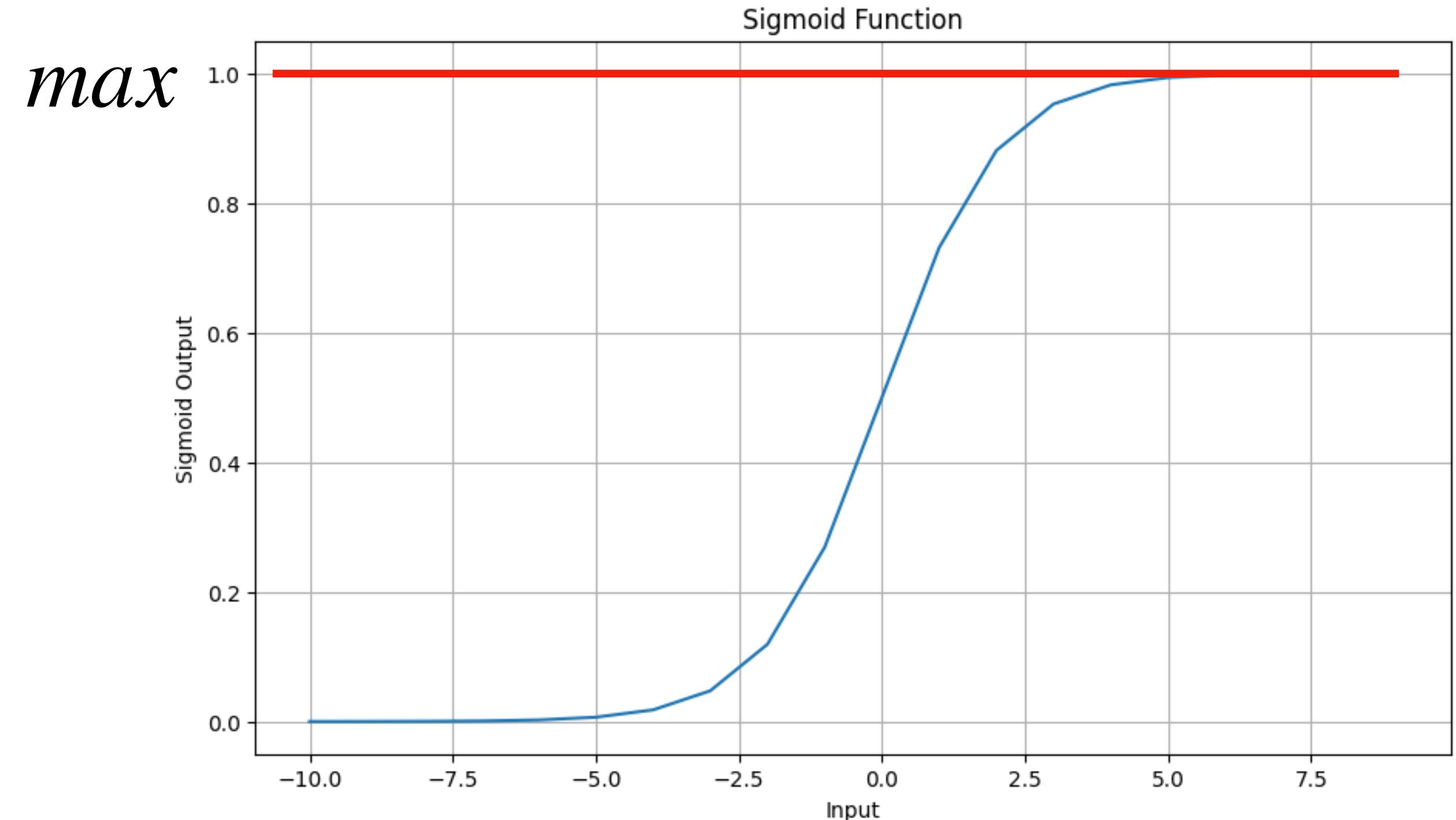
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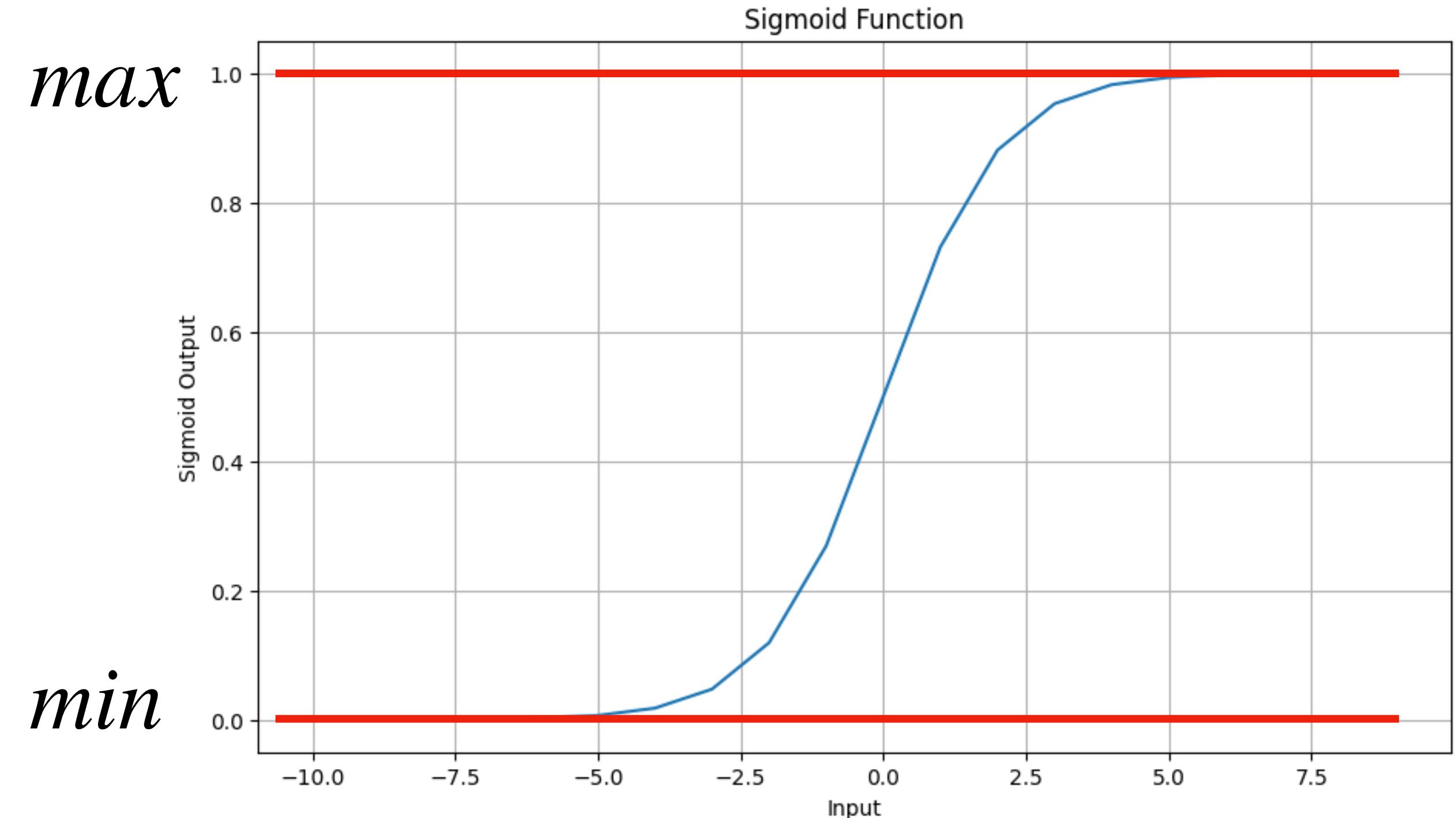
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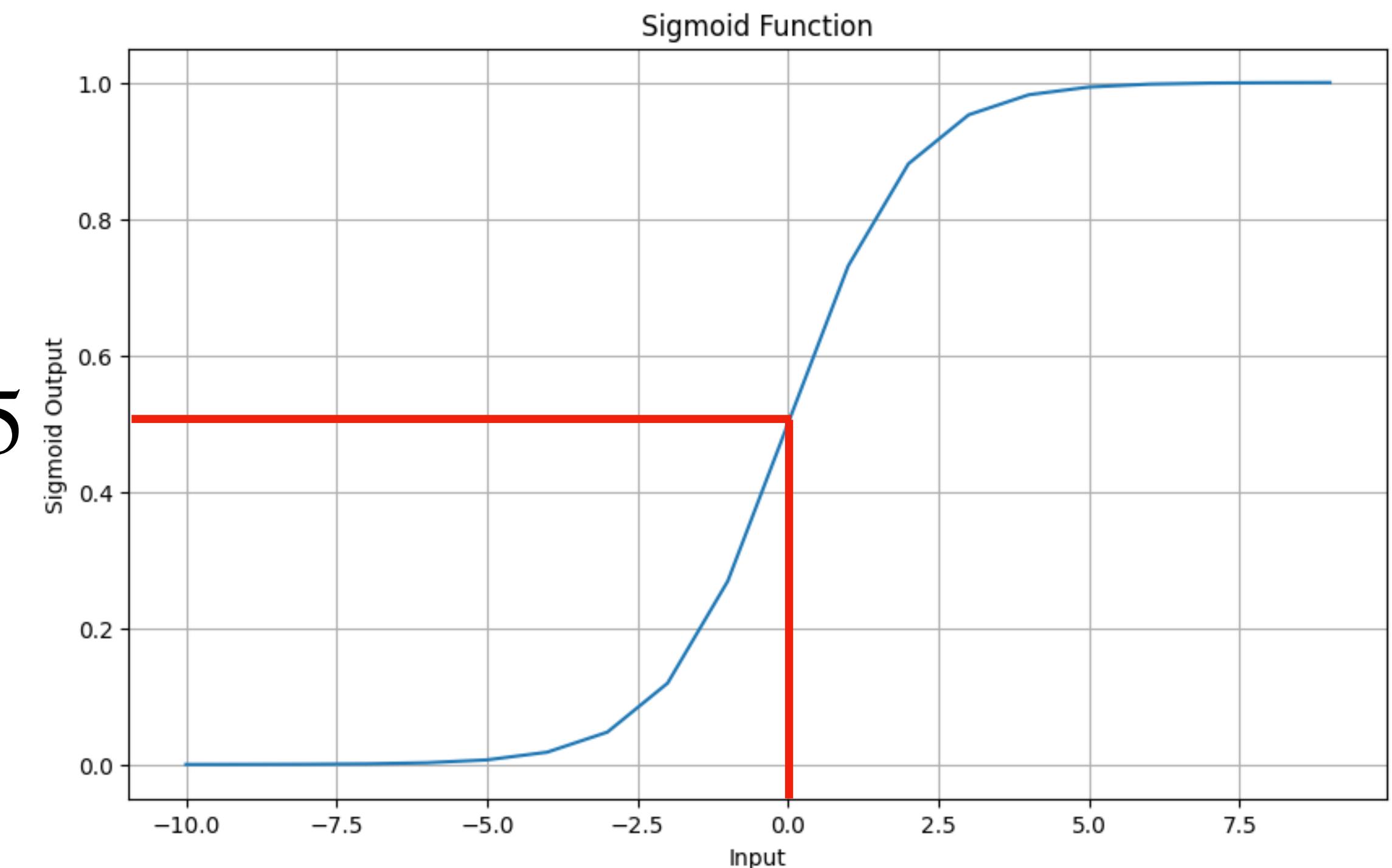


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$$mid = 0.5$$

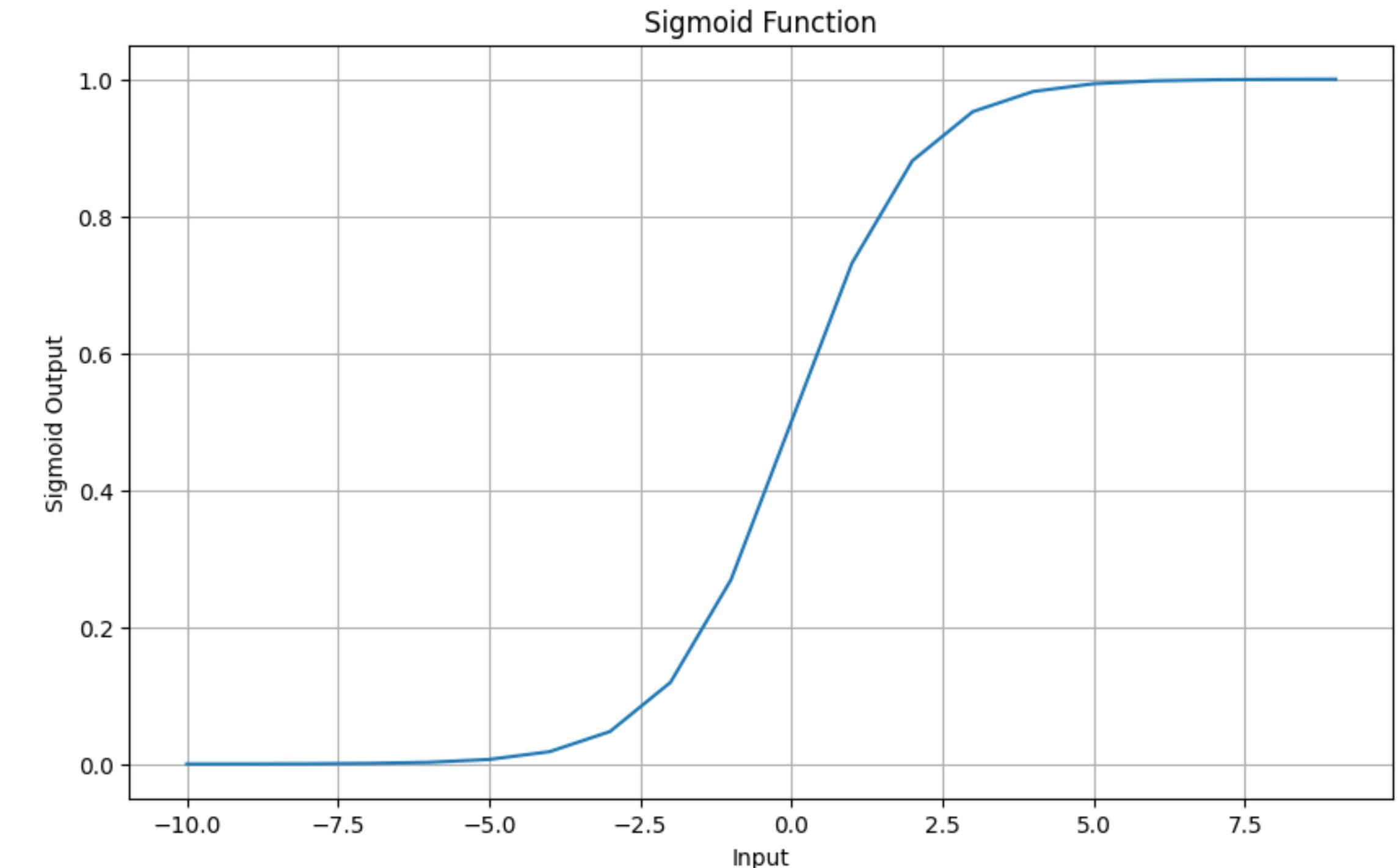


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- Linear regression predicts **unbounded** real values as  $\hat{y} = \theta_0 + \theta_1 \cdot x$
- But we need probabilities in  $[0, 1]$



# Logistic Regression

- Wrap the linear regression equation in a Sigmoid function
- Logistics regression models the probability of the positive class

$$\mathbb{P}(Y = 1 | X = x) = \sigma(\theta_0 + \theta_1 \cdot x) = \frac{1}{1 + e^{-(\theta_0 + \theta_1 \cdot x)}}$$

The decision boundary is the hyperplane where  $\mathbb{P}(Y = 1 | X = x) = 0.5$ , which occurs when  $\theta_0 + \theta_1 \cdot x = 0$

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**Assume that threshold = 0.5**

If  $\theta_0 + \theta_1 \cdot x \geq 0$ , classify as “positive class”  
Why?

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Because  $\sigma(k \geq 0) \geq 0.5$

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Why?

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Why?

Because  $\sigma(k \leq 0) \leq 0.5$

# Logistic Regression

Model:

$$\hat{y} = \sigma(\theta_0 + \theta_1 \cdot x)$$

Loss:

$$\ell(\theta) = -\frac{1}{m} \sum_{i=1}^m y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

# Logistic Regression

How do we train this?

## Maximum Likelihood Estimation

Maximum Likelihood Estimation (MLE) is a principled method for **estimating the parameters of a statistical model**.

**Key Idea** - Choose parameters that make the observed data **most probable**.

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Maximum Likelihood Estimation (MLE) is a principled method for **estimating the parameters of a statistical model**.

**Key Idea** - Choose parameters that make the observed data **most probable**.

Given some dataset  $D$  and a model with parameters  $\theta$

$$\hat{\theta}_{MLE} = \arg \max_{\theta} \mathbb{P}(D | \theta)$$

# Logistic Regression

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Probability that we observe training dataset  $D$ , given that the model has parameters  $\theta$

# Logistic Regression

How do we train this?

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Given some dataset  $D$  and a model with parameters  $\theta$

$$\hat{\theta}_{MLE} = \arg \max_{\theta} \mathbb{P}(D | \theta)$$

Find  $\theta$  such that this probability is maximized

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How do we train this?

## Maximum Likelihood Estimation

**Key Idea - Choose parameters that make the observed data most probable.**

Given some dataset  $D$  and a model with parameters  $\theta$

$$\hat{\theta}_{MLE} = \arg \max_{\theta} \mathbb{P}(D | \theta)$$

Under what parameter values would we have been **most likely to observe exactly the data we did observe?**

# Logistic Regression

How do we train this?

## Maximum Likelihood Estimation

What we want to find:

$$\hat{\theta}_{MLE} = \arg \max_{\theta} \mathbb{P}(D | \theta)$$

**Probability:**

$\mathbb{P}(D | \theta)$  - Given **fixed parameters**  $\theta$ ,  
what is the probability of observing data  $D$ ?  
This is a function of  $D$  with  $\theta$  fixed.

# Logistic Regression

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**Likelihood:**

$L(\theta | D) = \mathbb{P}(D | \theta)$  - Given fixed observed data  $D$ , how **likely** are different parameter values  $\theta$ ?  
This is a function of  $\theta$  with  $D$  fixed.

# Logistic Regression

## Probability vs Likelihood

### Coin Flips

Suppose you flip a coin **10 times** and get **7 heads**.

# Logistic Regression

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**Probability Perspective:** If  $\theta = 0.5$  (fair coin), what's  $\mathbb{P}(7 \text{ heads in 10 flips} \mid \theta)$ ?

Answer:  $\mathbb{P}(X = 7 \mid \theta = 0.5) = \binom{10}{7} \cdot 0.5^7 \cdot (1 - 0.5)^{10-7} \approx 0.117$

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$$L(\theta = 0.5 \mid X = 7) = 0.117$$

$$L(\theta = 0.7 \mid X = 7) = 0.267 \text{ (higher)}$$

$$L(\theta = 0.3 \mid X = 7) = 0.009 \text{ (lower)}$$

# Logistic Regression

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## Likelihood Function

For **independent** observations (rows of data)  $D = \{x^{(1)}, x^{(2)}, x^{(3)}, \dots, x^{(m)}\}$ ,  
the likelihood is the **product** of individual probabilities

$$L(\theta | D) = \mathbb{P}(D | \theta) = \prod_{i=1}^m \mathbb{P}(x^{(i)} | \theta)$$

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But, products are numerically **unstable** and difficult to differentiate  
So, we take *log* on both sides to convert products to sums

# Logistic Regression

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$$\log(L(\theta | D)) = \sum_{i=1}^m \log(\mathbb{P}(x^{(i)} | \theta))$$

Using properties of  $\log$ :

$$\begin{aligned}\log(a^b) &= b \cdot \log(a) \\ \log(ab) &= \log(a) + \log(b)\end{aligned}$$

# Logistic Regression

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**For logistic regression**

Input Features:  $x \in \mathbb{R}^m$

Binary Labels:  $y \in \{0, 1\}$

Training Data:  $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), (x^{(3)}, y^{(3)}), \dots, (x^{(m)}, y^{(m)})\}$

# Logistic Regression

$$\mathbb{P}(Y = 1 \mid X = x; \theta) = \sigma(\theta_0 + \theta_1 \cdot x)$$

# Logistic Regression

$$\mathbb{P}(Y = 1 | X = x; \theta) = \sigma(\theta_0 + \theta_1 \cdot x)$$

Each label  $y_i$  follows a **Bernoulli Distribution** with parameter

$$p_i = \mathbb{P}(Y = 1 | x_i)$$

# Logistic Regression

## Quick Aside: Bernoulli Distribution

Bernoulli Distribution models a single binary outcome

Is  $\mathbb{P}(X = \text{success}) = p$  and  
 $\mathbb{P}(X = \text{failure}) = q = (1 - p)$

Then probability mass function  $P$  is

$$P(X = x) = p^x \cdot (1 - p)^{1-x}$$

# Logistic Regression

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When  $y = 1 \rightarrow p^1(1 - p)^0 = p$

When  $y = 0 \rightarrow p^0(1 - p)^1 = (1 - p)$

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$$\text{When } y = 1 \rightarrow p^1(1 - p)^0 = p$$

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# Logistic Regression

For a **single** observation  $(x^{(i)}, y^{(i)})$

**Probability** of observing  $y^{(i)}$  given you have seen input data  $x^{(i)}$  and  $\theta$

$$\mathbb{P}(y^{(i)} | x^{(i)}; \theta) = p_i^{y^{(i)}} (1 - p_i)^{1-y^{(i)}}$$

Where  $p_i = \sigma(\theta_0 + \theta_1 \cdot x)$

# Logistic Regression

For the **entire dataset**  $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), (x^{(3)}, y^{(3)}), \dots, (x^{(m)}, y^{(m)})\}$

Assuming observations are **independent**

**Likelihood** is the product of all individual probabilities

$$L(\theta | D) = \prod_{i=1}^m \mathbb{P}(y^{(i)} | x^{(i)}; \theta) = \prod_{i=1}^m p_i^{y^{(i)}} (1 - p_i)^{1 - y^{(i)}}$$

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We want to **maximize** likelihood

$$\hat{\theta}_{MLE} = \arg \max_{\theta} \mathbb{P}(D | \theta)$$

# Logistic Regression

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$$L(\theta | D) = \prod_{i=1}^m p_i^{y^{(i)}} (1 - p_i)^{1-y^{(i)}}$$

$$\log(L(\theta)) = \log(\prod_{i=1}^m p_i^{y^{(i)}} (1 - p_i)^{1-y^{(i)}})$$

Using properties of log:

$$\log(a^b) = b \cdot \log(a)$$

$$\log(ab) = \log(a) + \log(b)$$

$$\log(L(\theta)) = \sum_{i=1}^m \log(p_i^{y^{(i)}} (1 - p_i)^{1-y^{(i)}})$$

$$\log(L(\theta)) = \sum_{i=1}^m y^{(i)} \log(p_i) + (1 - y^{(i)}) \log(1 - p_i)$$

# Logistic Regression

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$$\log(L(\theta)) = \sum_{i=1}^m y^{(i)} \log(p_i) + (1 - y^{(i)}) \log(1 - p_i)$$

This is called the **log-likelihood** function for logistic regression

# Logistic Regression

$$\log(L(\theta)) = \sum_{i=1}^m y^{(i)} \log(p_i) + (1 - y^{(i)}) \log(1 - p_i)$$

This is called the **log-likelihood** function for logistic regression

Remember we want to **maximize** likelihood

But when we deal with “loss” functions and gradient descent, we want to **minimize** the loss

# Logistic Regression

$$\ell(\theta) = - \sum_{i=1}^m y^{(i)} \log(p_i) + (1 - y^{(i)}) \log(1 - p_i)$$

Solution: Minimize **negative** likelihood

# Logistic Regression

$$\ell(\theta) = - \sum_{i=1}^m y^{(i)} \log(p_i) + (1 - y^{(i)}) \log(1 - p_i)$$

Solution: Minimize **negative** likelihood

Remember that  $p_i$  is the predicted output where

$$p_i = \sigma(\theta_0 + \theta_1 \cdot x)$$

# Logistic Regression

$$\ell(\theta) = -\frac{1}{m} \sum_{i=1}^m y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

Binary Cross Entropy Loss

# Logistic Regression

$$\ell(\theta) = -\frac{1}{m} \sum_{i=1}^m y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

When  $y^{(i)} = 1$ , i.e., actual positive

$$\ell(\theta) = - \log(\hat{y}^{(i)})$$

When  $y^{(i)} = 0$ , i.e., actual negative

$$\ell(\theta) = - \log(1 - \hat{y}^{(i)})$$

# Logistic Regression

$$\ell(\theta) = -\frac{1}{m} \sum_{i=1}^m y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

When  $y^{(i)} = 1$ , i.e., actual positive

$$\ell(\theta) = -\log(\hat{y}^{(i)}) \quad \begin{array}{l} \text{If } \hat{y}^{(i)} = 1, \text{ Loss} = 0 \\ \text{If } \hat{y}^{(i)} = 0, \text{ Loss} = +\infty \end{array}$$

When  $y^{(i)} = 0$ , i.e., actual negative

$$\ell(\theta) = -\log(1 - \hat{y}^{(i)}) \quad \begin{array}{l} \text{If } \hat{y}^{(i)} = 0, \text{ Loss} = 0 \\ \text{If } \hat{y}^{(i)} = 1, \text{ Loss} = +\infty \end{array}$$

# Logistic Regression

## Finding $\theta$

$$\ell(\theta) = -\frac{1}{m} \sum_{i=1}^m y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

Find partial derivative

To simplify, lets find the derivative for a **single** sample

# Logistic Regression

## Finding $\theta$

$$\ell(\theta) = y \log(\hat{y}) + (1 - y) \log(1 - \hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = \theta_0 + \theta_1 x$$

Want to find  $\frac{\partial \ell}{\partial \theta}$

Using Chain Rule

$$\frac{\partial \ell}{\partial \theta} = \frac{\partial \ell}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial z}{\partial \theta}$$

# Logistic Regression

## Finding $\theta$

Summing over all samples

$$\frac{\partial \ell}{\partial \theta} = \frac{1}{m} \sum_{i=1}^m x^{(i)} \cdot (\hat{y}^{(i)} - y^{(i)})$$

In matrix form

$$\nabla_{\theta}(\ell(\theta)) = \frac{1}{m} X^T (\hat{Y} - Y)$$

# Logistic Regression

## Summary

Model:

$$\hat{y} = \sigma(\theta_0 + \theta_1 x)$$

Loss:

$$\ell(\theta) = -\frac{1}{m} \sum_{i=1}^m y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

Gradient:

$$\nabla_{\theta}(\ell(\theta)) = \frac{1}{m} X^T (\hat{Y} - Y)$$

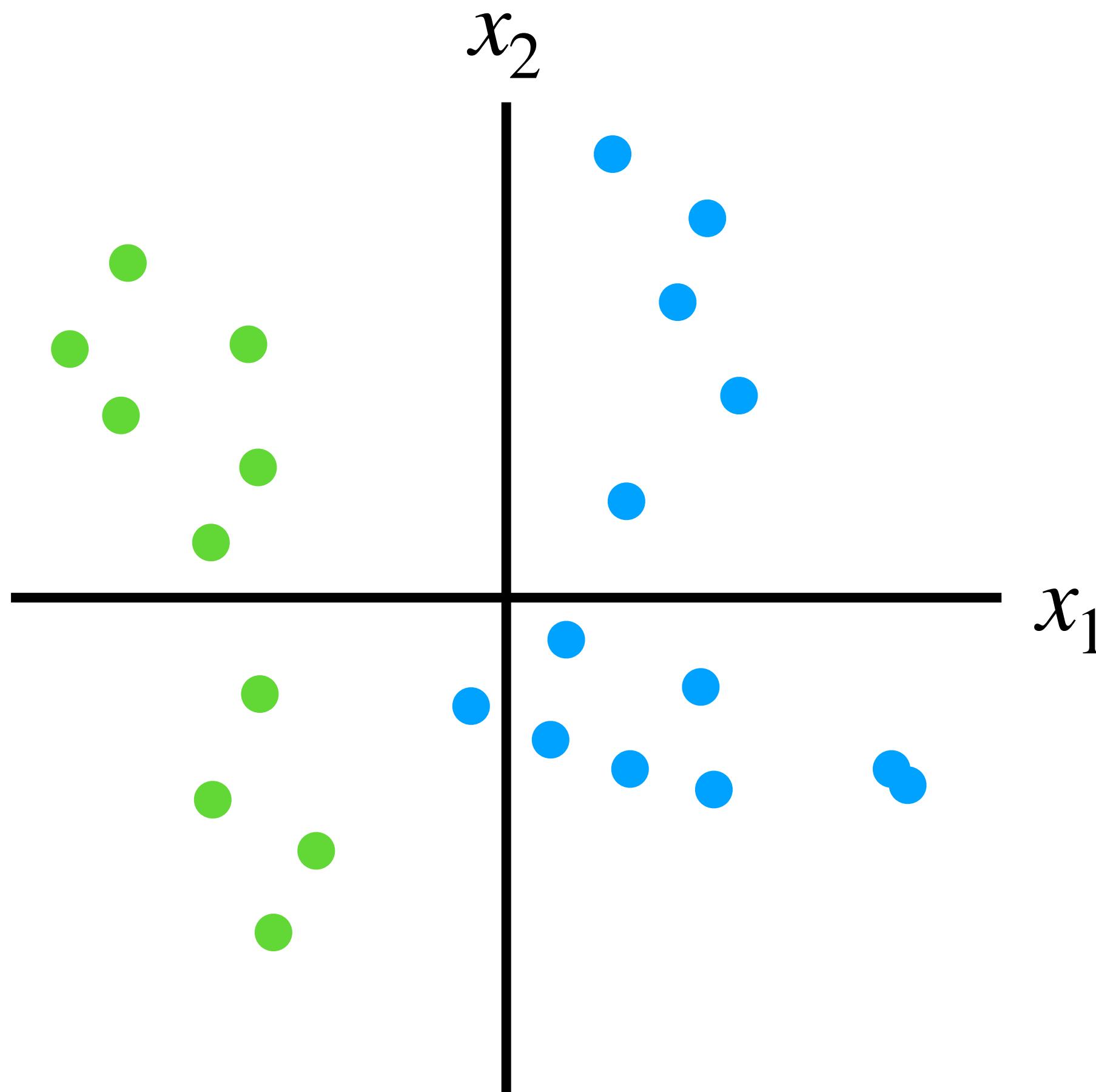
# Logistic Regression

## Summary

Why not MSE?

# Logistic Regression

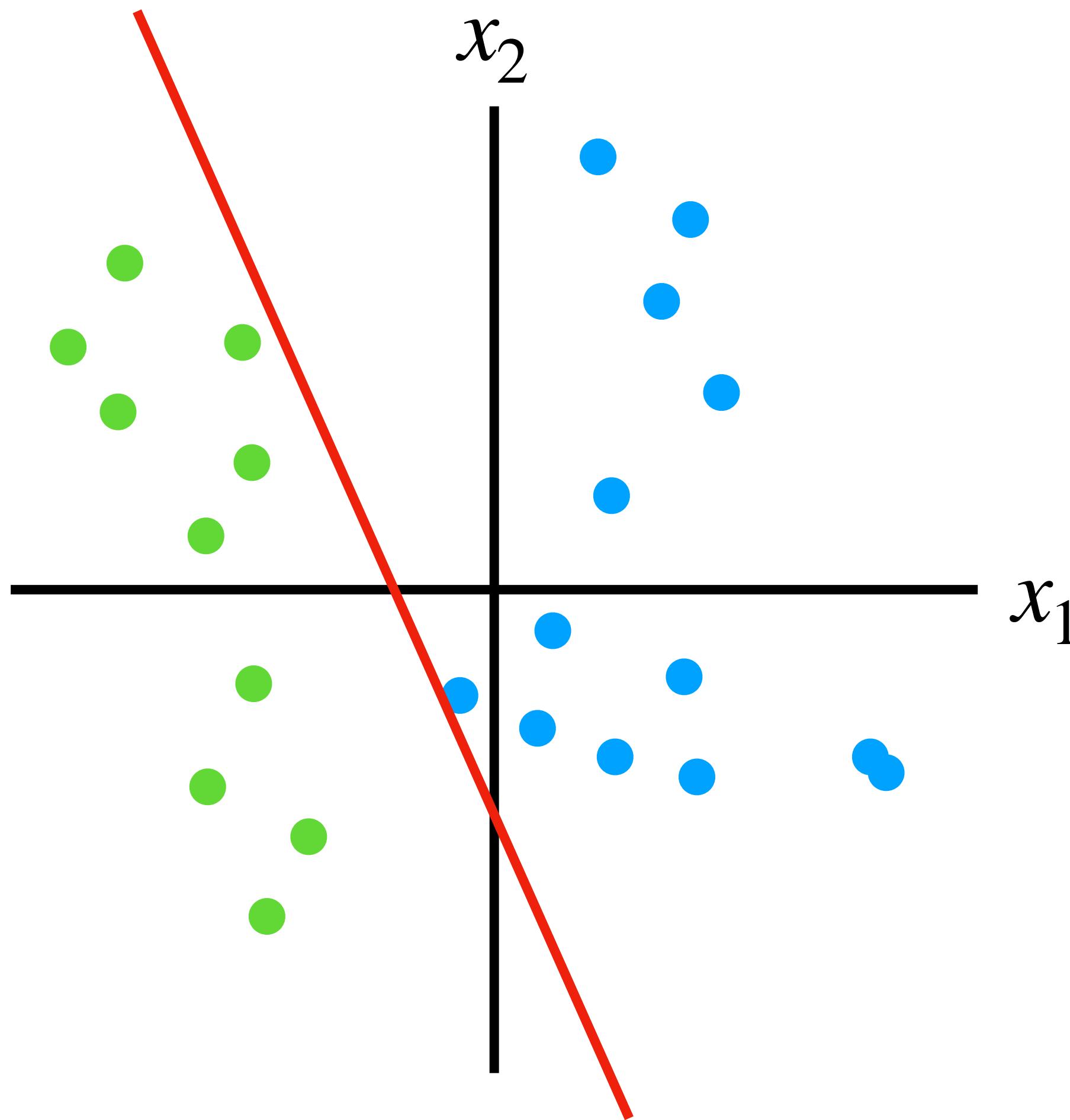
## Summary



# Logistic Regression

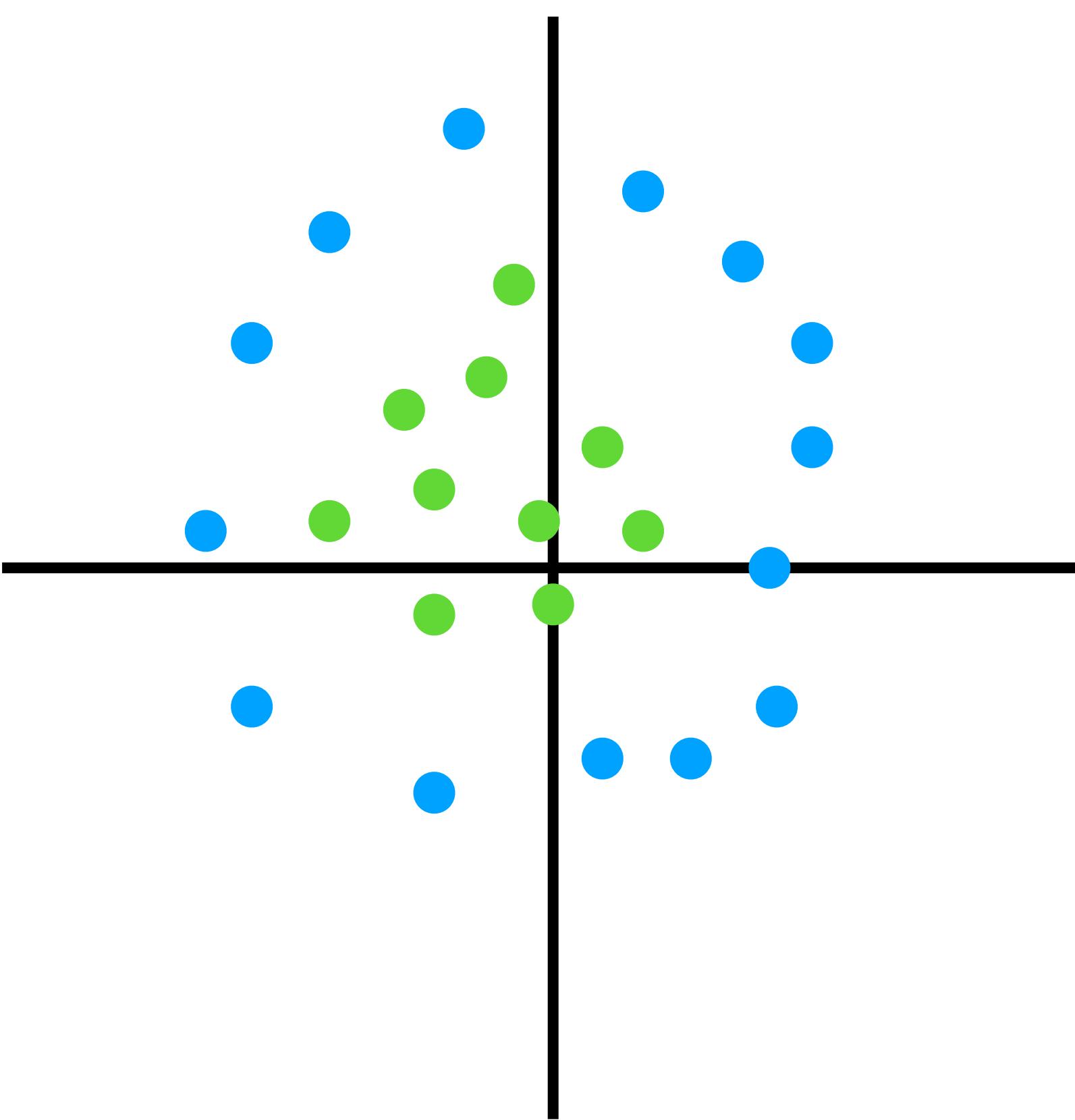
## Summary

$$\hat{y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$



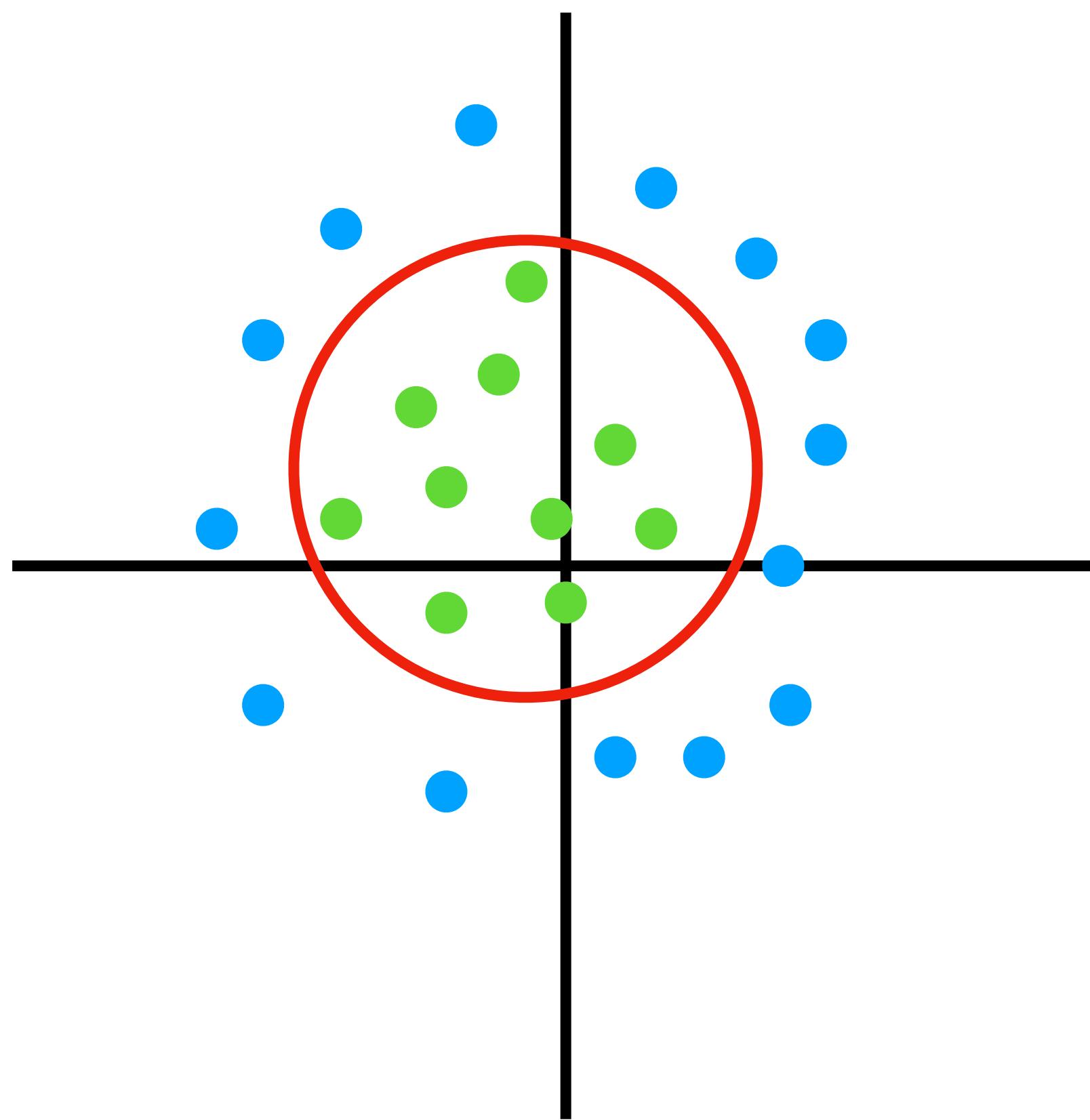
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# Logistic Regression

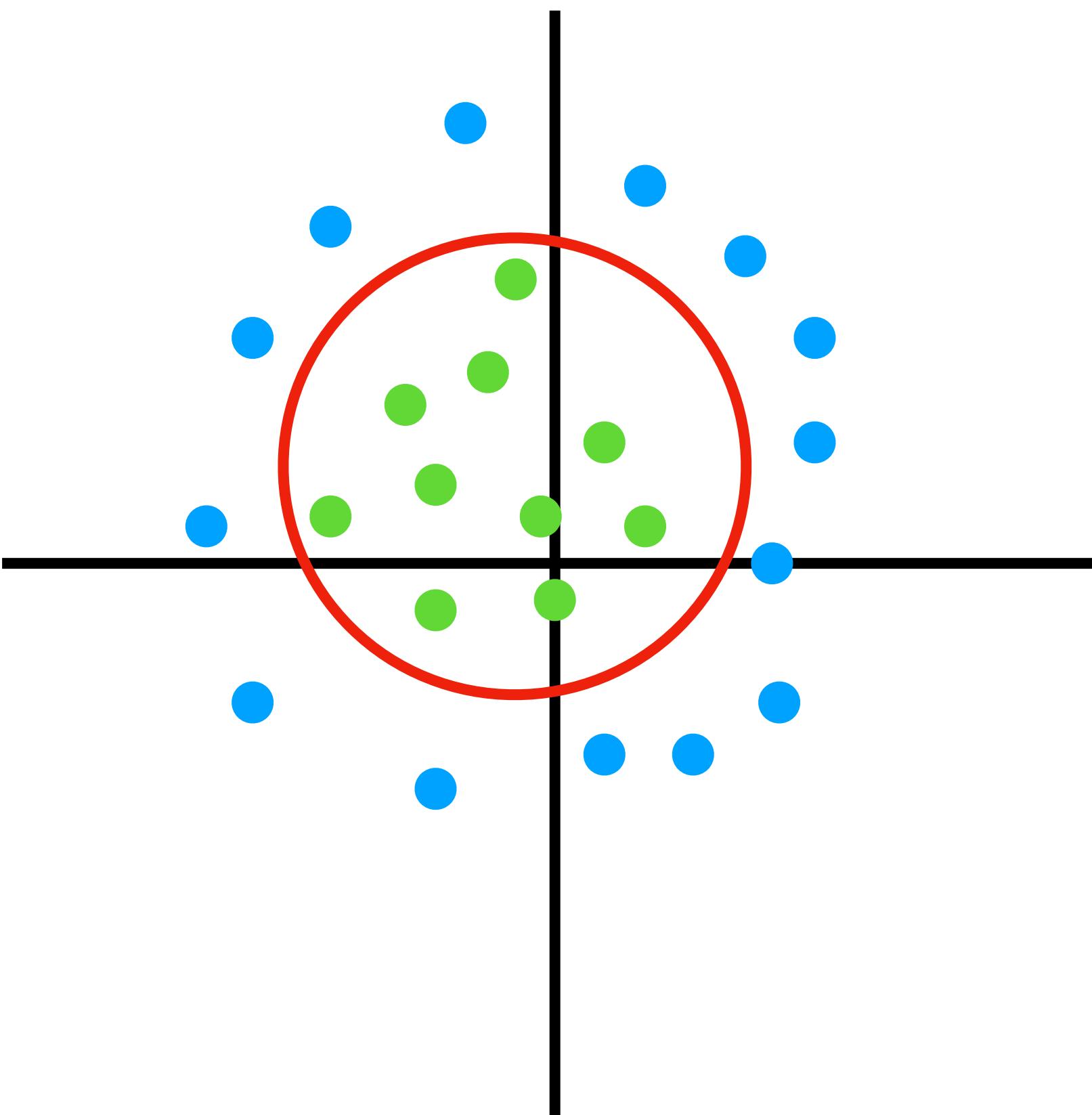
## Summary



# Logistic Regression

## Summary

$$x_1^2 + x_2^2 = r^2$$

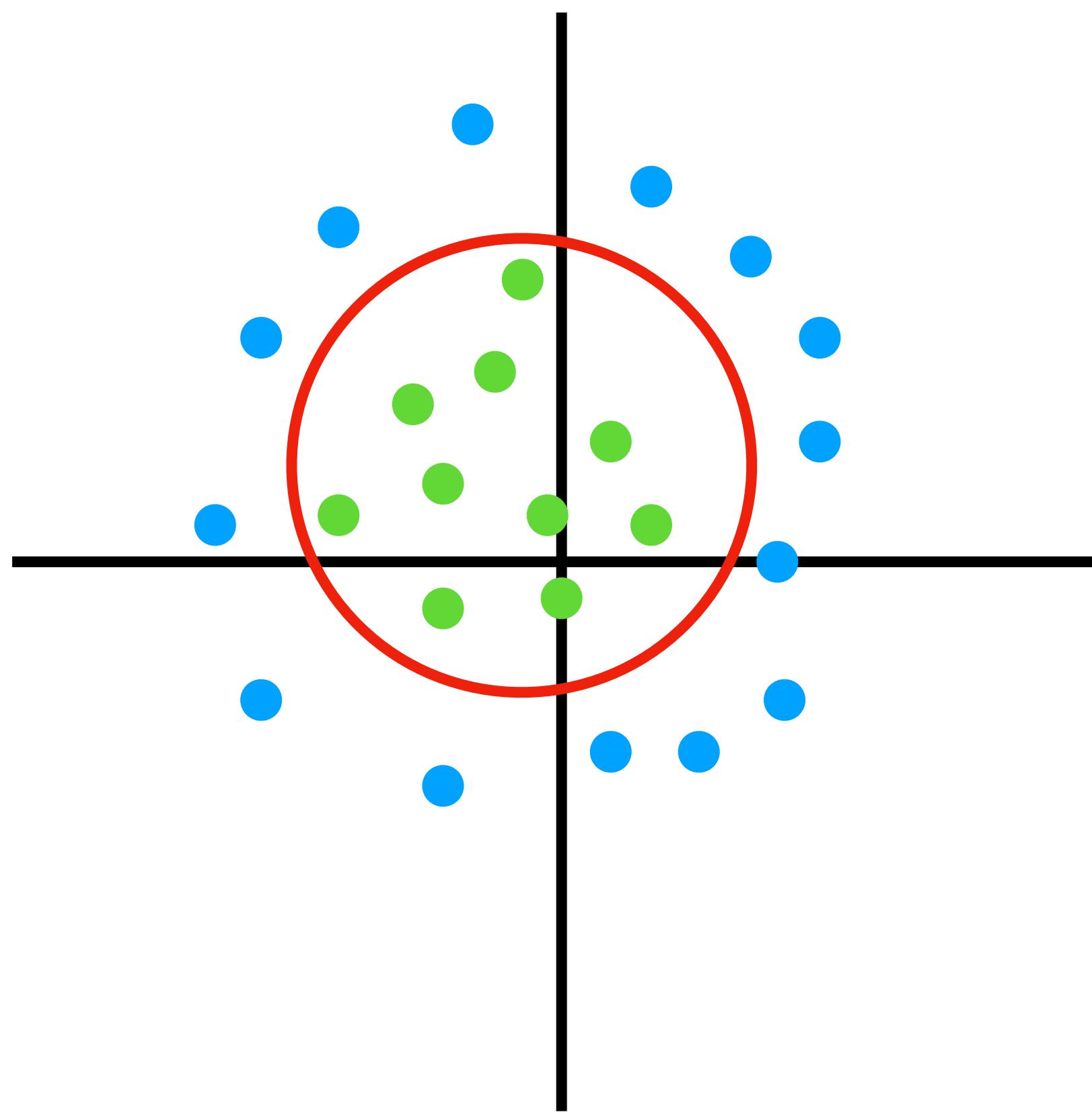


# Logistic Regression

## Summary

$$x_1^2 + x_2^2 = r^2$$

$$\theta_1^2(x_1^2 + x_2^2) = \theta_0^2$$



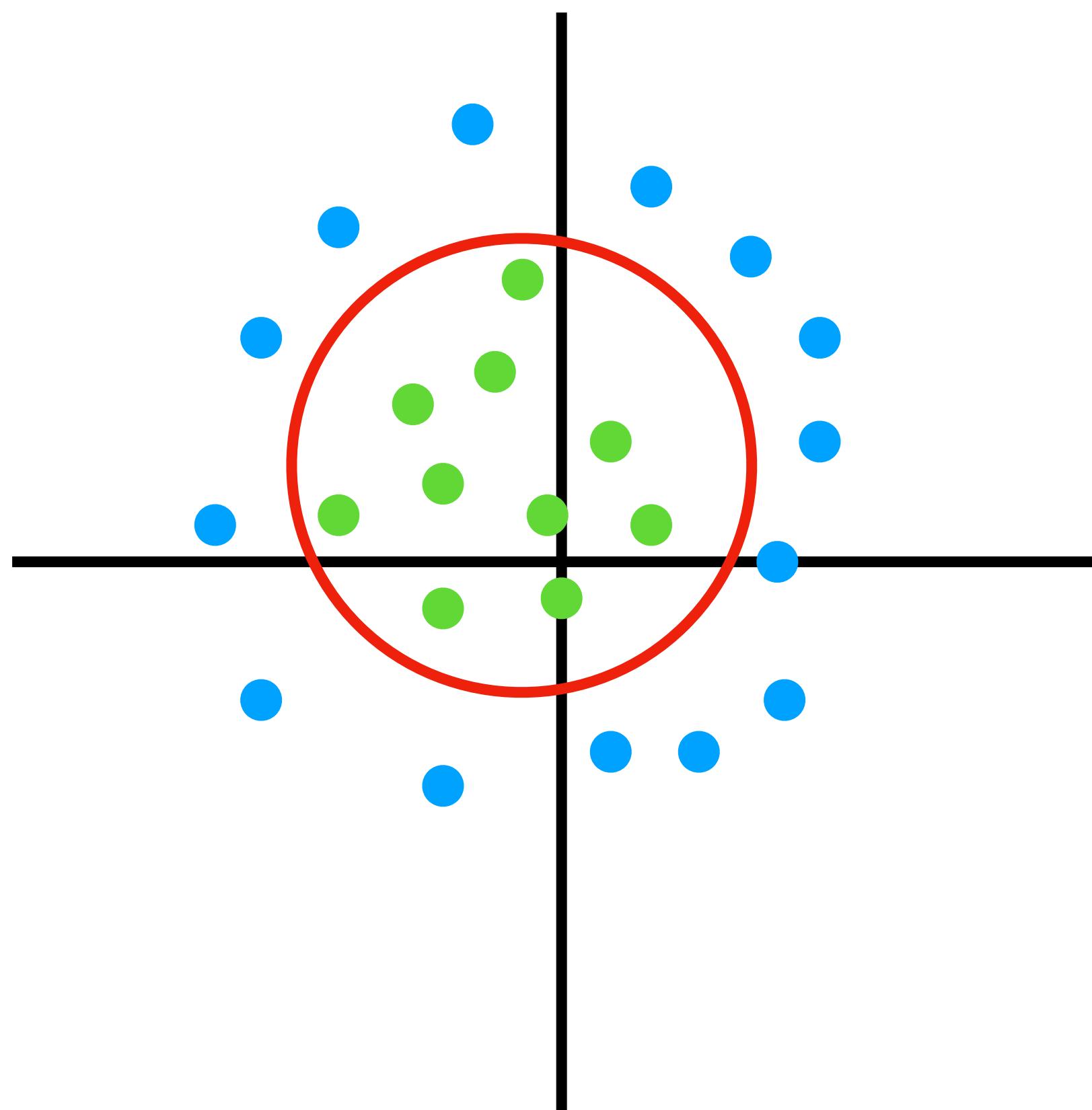
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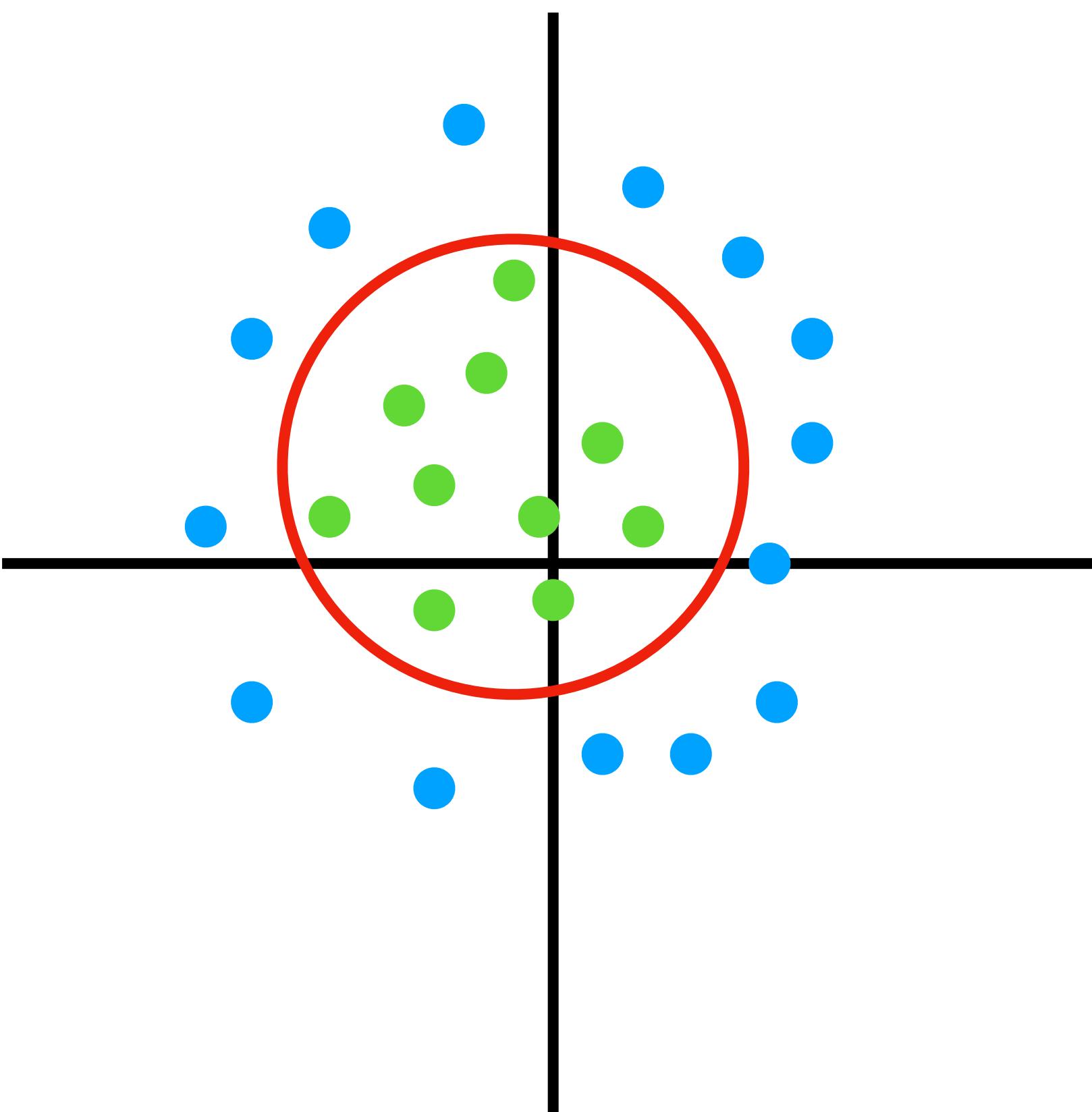
$$\theta_1^2(x_1^2 + x_2^2) = \theta_0^2$$

$$\sqrt{\theta_1^2(x_1^2 + x_2^2)} = \sqrt{\theta_0^2}$$



# Logistic Regression

## Summary



$$x_1^2 + x_2^2 = r^2$$

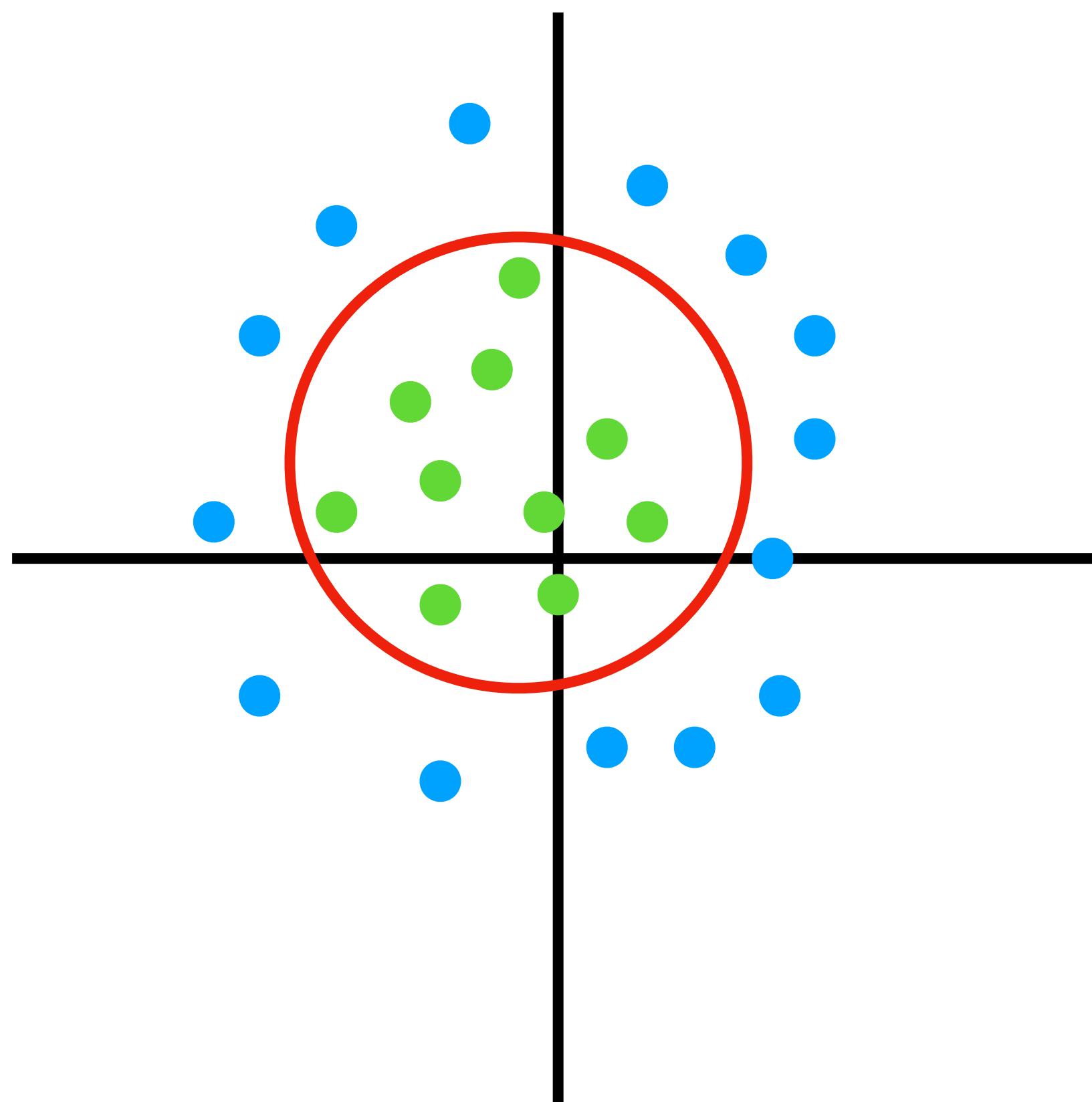
$$\theta_1^2(x_1^2 + x_2^2) = \theta_0^2$$

$$\sqrt{\theta_1^2(x_1^2 + x_2^2)} = \sqrt{\theta_0^2}$$

$$\theta_1 \sqrt{(x_1^2 + x_2^2)} = \theta_0$$

# Logistic Regression

## Summary



$$x_1^2 + x_2^2 = r^2$$

$$\theta_1^2(x_1^2 + x_2^2) = \theta_0^2$$

$$\sqrt{\theta_1^2(x_1^2 + x_2^2)} = \sqrt{\theta_0^2}$$

$$\theta_1\sqrt{(x_1^2 + x_2^2)} = \theta_0$$

$$\hat{y} = \theta_1\sqrt{(x_1^2 + x_2^2)} - \theta_0$$

# Next Class

- More classification algorithms