Northeastern University
Khoury College of
Computer Sciences

~~Ensemble Learning &~~ Deep Learning & Conv Networks,

DS 4400 | Machine Learning and Data Mining I

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Spring 2026

Monday | March 16th, 2026

Today’s Outline

- Deep Learning
- Convolutional Neural Networks

Deep Learning

Composing Functions

Model 1: $w_1 x + b = y_1$

Model 2: $w_2(y_1) + b$

$$x = [x_1 \quad x_2 \quad x_3]$$

Model 3: $x \cdot (w_1 w_2) + (w_1 \cdot w_2 + b) \cdot b$, with $w_3 = w_1 w_2$

Linear Reg.

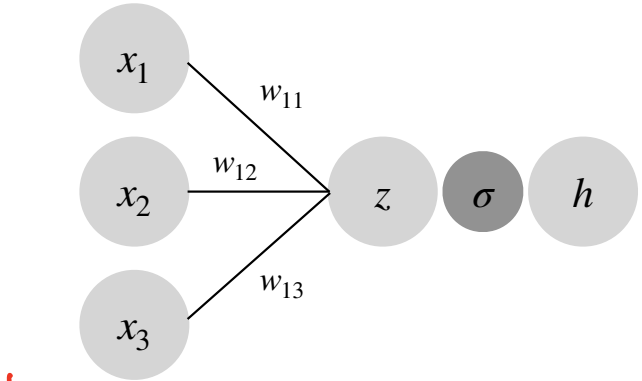
$$z = W_1 x + b_1$$

Logistic Reg.

$$h = \sigma(z)$$

$$\hat{y} = W_2 h + b_2$$

$$W_1 = \begin{bmatrix} w_{11} \\ w_{12} \\ w_{13} \end{bmatrix} \qquad W_2 = [w_{21}]$$



Deep Learning

Composing Functions

$$x = [x_1 \quad x_2 \quad x_3]$$

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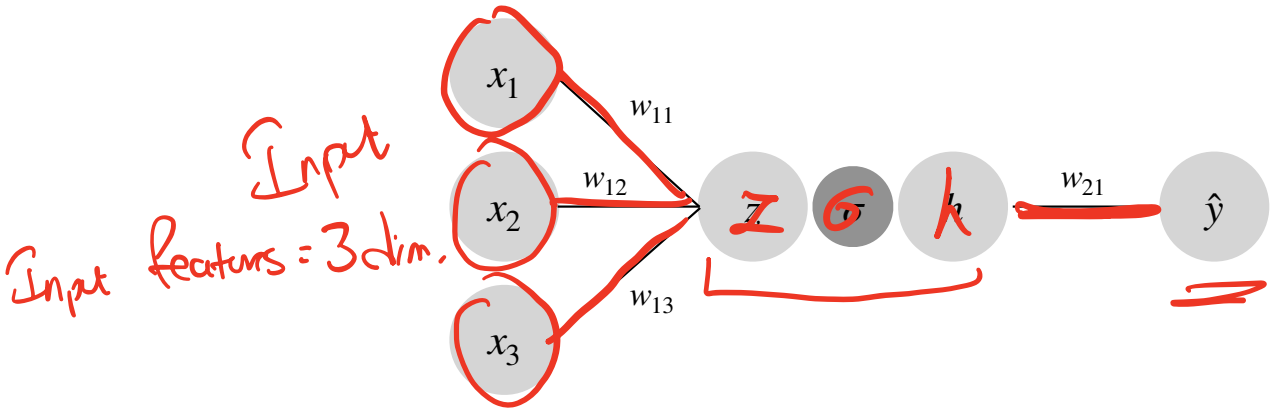

Deep Learning

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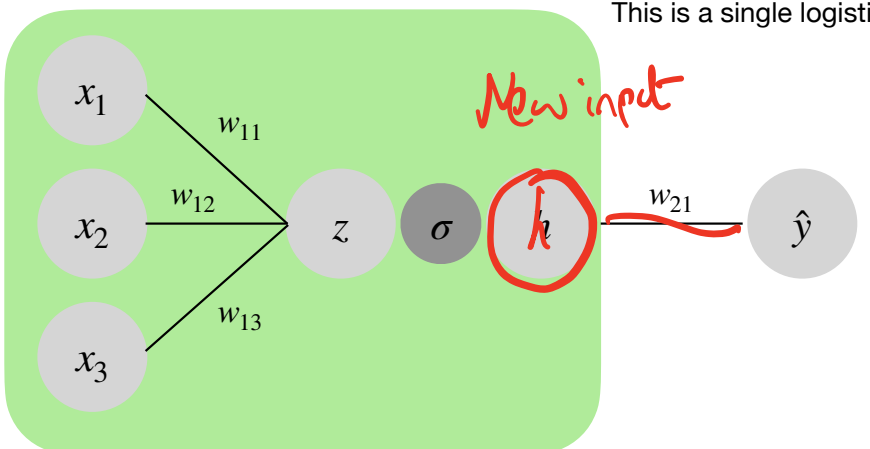
$$\hat{y} = W_2 h + b_2$$

This is a single logistic regression unit



Deep Learning

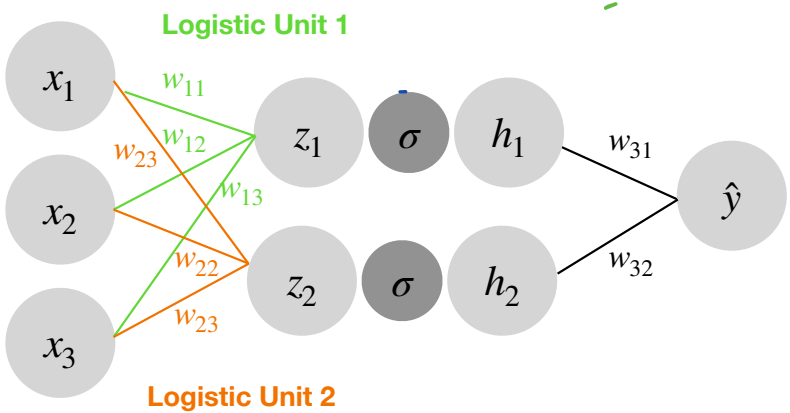
Composing Functions

$$h = \sigma(W_1 x + b_1)$$

$$\hat{y} = W_2 h + b_2$$

$$x = [x_1 \quad x_2 \quad x_3]$$

$$W_1 = \begin{bmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \\ w_{13} & w_{23} \end{bmatrix} \qquad W_2 = \begin{bmatrix} w_{31} \\ w_{32} \end{bmatrix}$$



Deep Learning

Composing Functions

$$x = [x_1 \quad x_2 \quad x_3]$$

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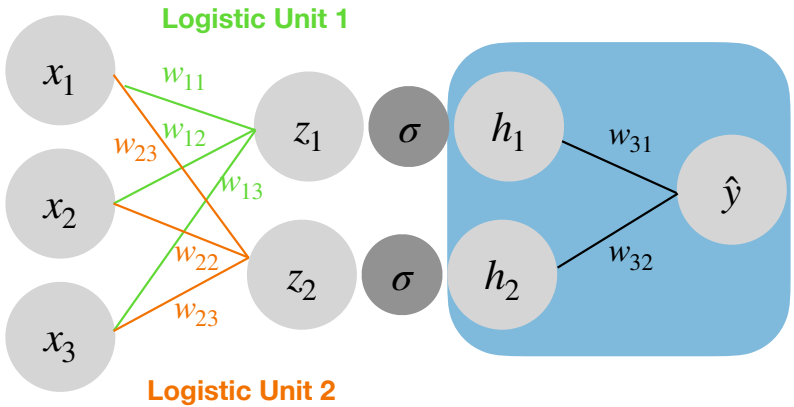
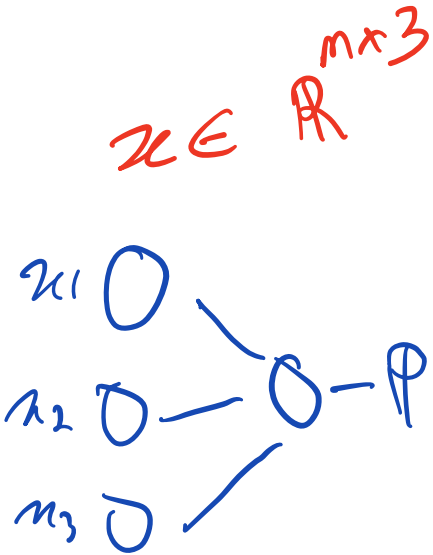
$$\hat{y} = W_2 h + b_2$$

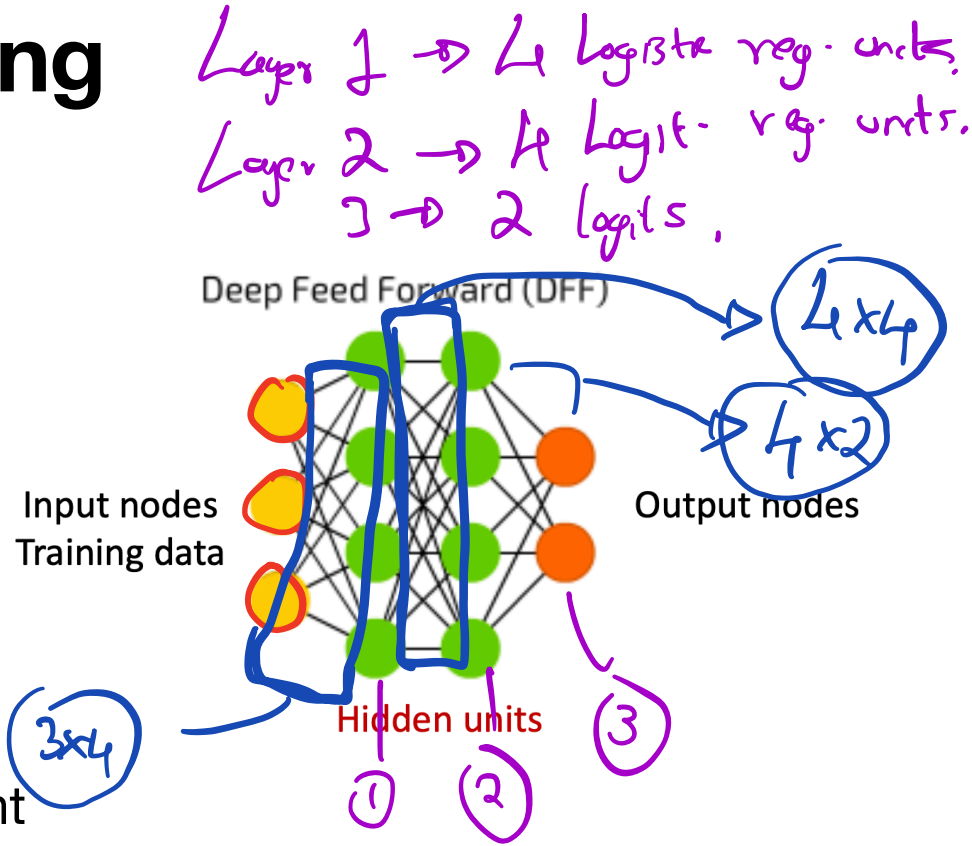


This is also a Logistic Unit!
It takes as **input** the outputs of the previous layer

Deep Learning

Neural Networks

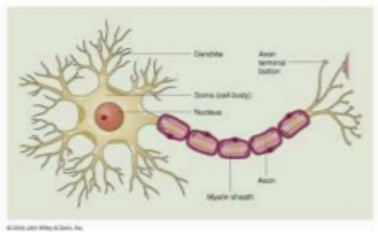


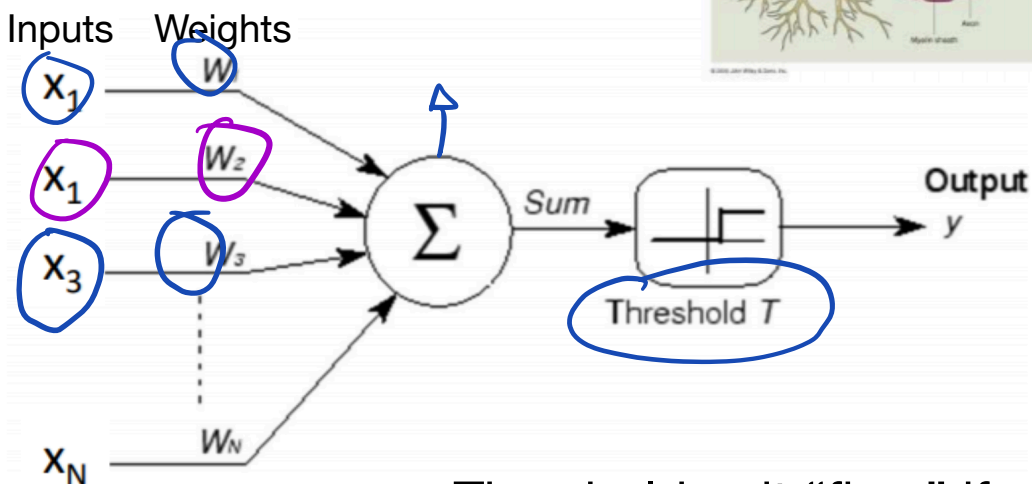


- Each link has a weight
- Each hidden unit has an activation function
- Training data is input in the left, output is generated on the right

Deep Learning

Perceptron





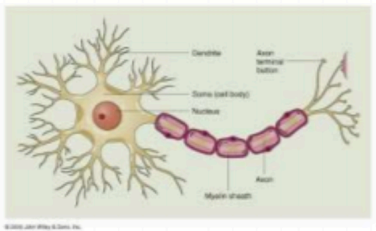
$$y = \begin{cases} 1 \text{ if } \sum_i w_i x_i \geq T \\ 0 \text{ else} \end{cases}$$

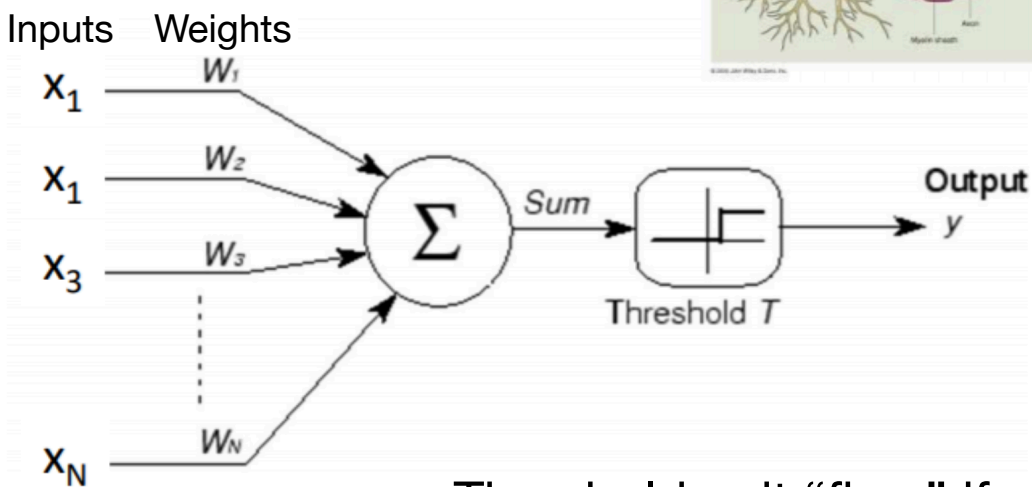
Threshold unit "fires" if weighted sum of inputs exceeds a thresholds

Deep Learning

Perceptron

$$\hat{y} = h(x) = sign(W^T x)$$





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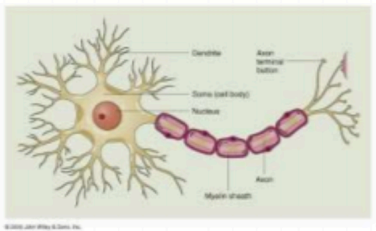
Threshold unit “fires” if weighted sum of inputs exceeds a thresholds

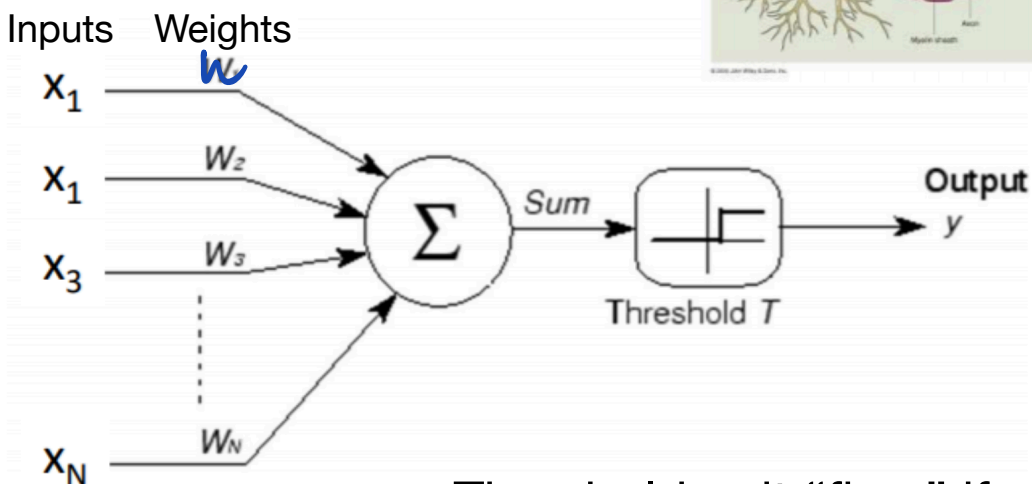
Deep Learning

Perceptron

$$\hat{y} = h(x) = sign(W^T x)$$

$$\theta_j \leftarrow \theta_j - \frac{1}{2}(h_w(x_i) - y_i) \cdot x_{ij}$$





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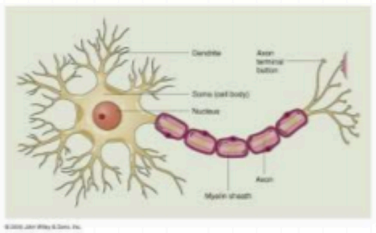
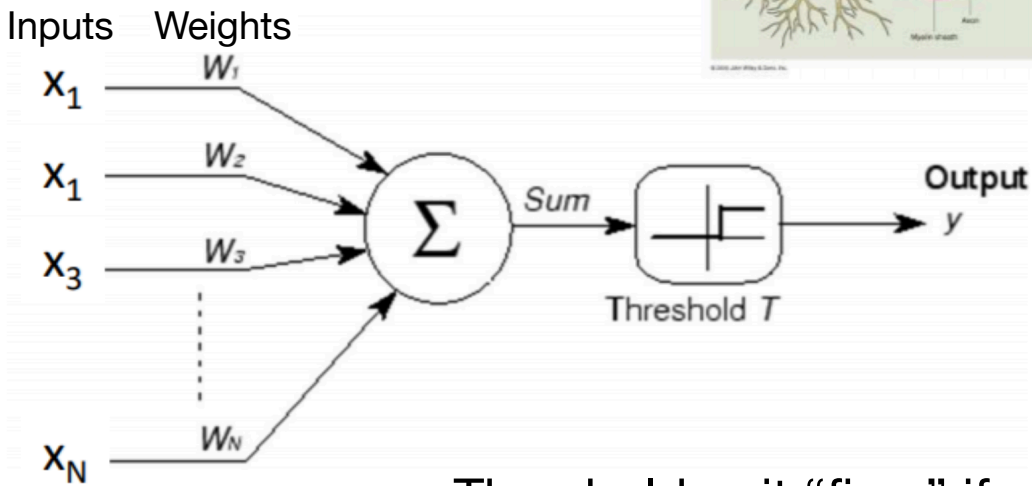


- If prediction matches label, do not change
- If not, change θ



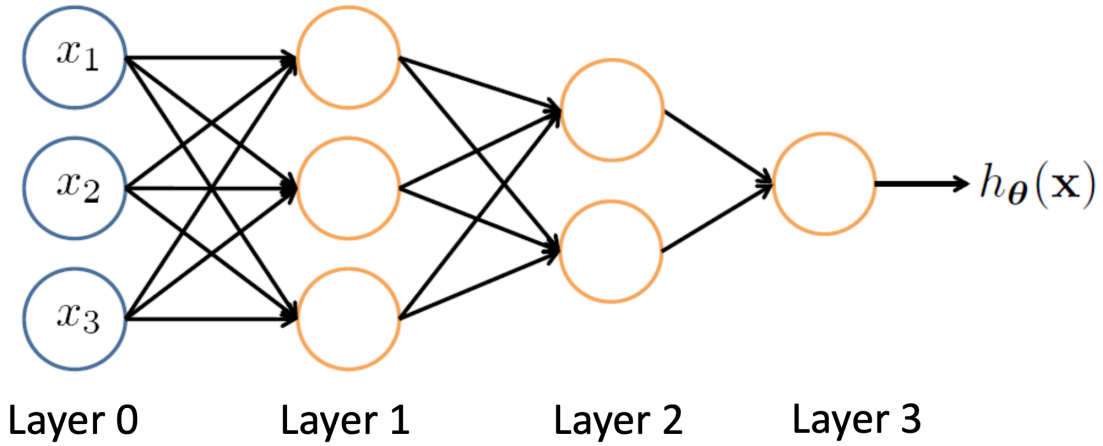
$$y = \begin{cases} 1 \; if \; \sum_i w_i x_i \geq T \\ 0 \; else \end{cases}$$

Threshold unit "fires" if weighted sum of inputs exceeds a thresholds

Deep Learning

Multi Layer Perceptron (MLP)

- MLP / Feed Forward Network / Fully Connected Network / Neural Network



Deep Learning

Multi Layer Perceptron (MLP)

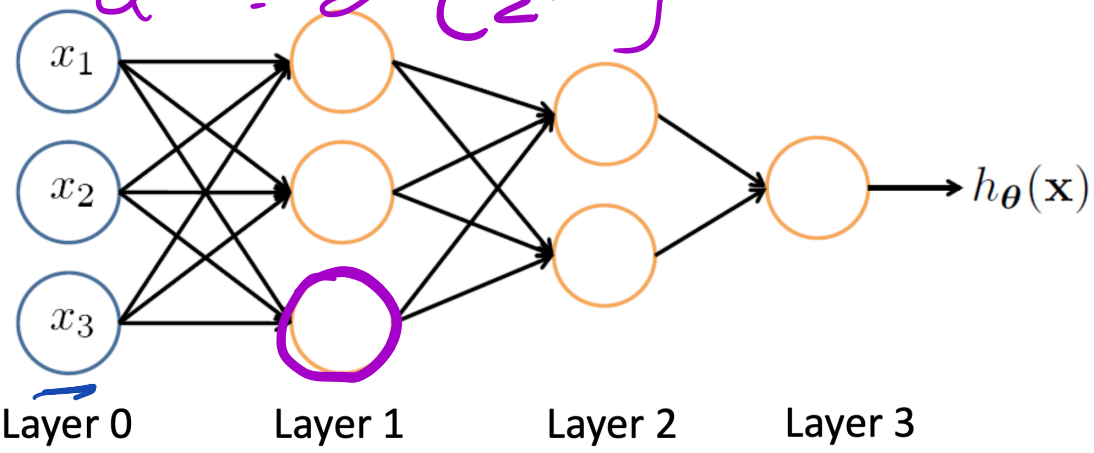
Number of layers $L = 3$

Computation:

$a^{[0]} = x$ (input)
For each layer $l = 1,2,...L$:

$z^{[l]} = W^{[l]}a^{[l-1]} + b^{[l]}$ (linear transform)
$a^{[l]} = \sigma^{[l]}(z^{[l]})$ (activation)

Final Output: $h_\theta(x) = \hat{y} = a^{[L]}$

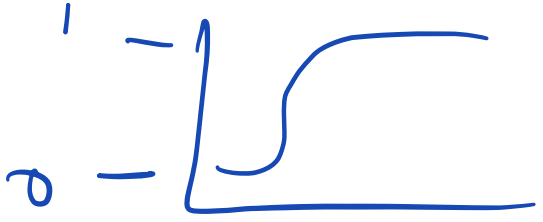


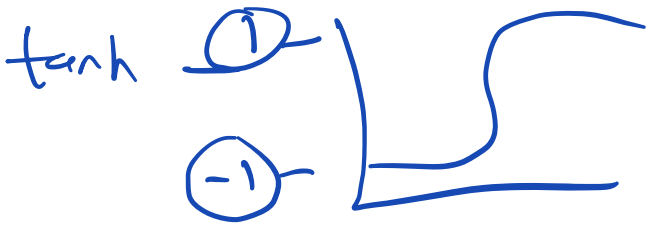
Deep Learning

Activation Functions

non linear function

- Why Activation Functions Matter
  - Introduce non-linearity (**essential for learning complex patterns**)
  - Control the range of outputs
  - Affect gradient flow during training





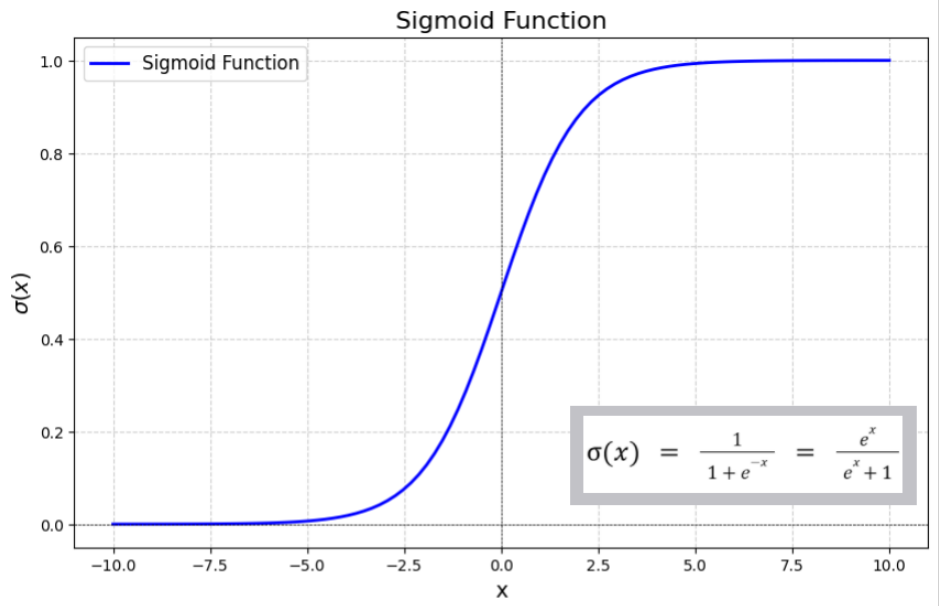
Deep Learning

Activation Functions

- Sigmoid Function

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Range: [0, 1]



Pros: Smooth, bounded, interpretable as probability

Cons: Vanishing gradients (saturates at extremes), not zero-centered, expensive exponential operation

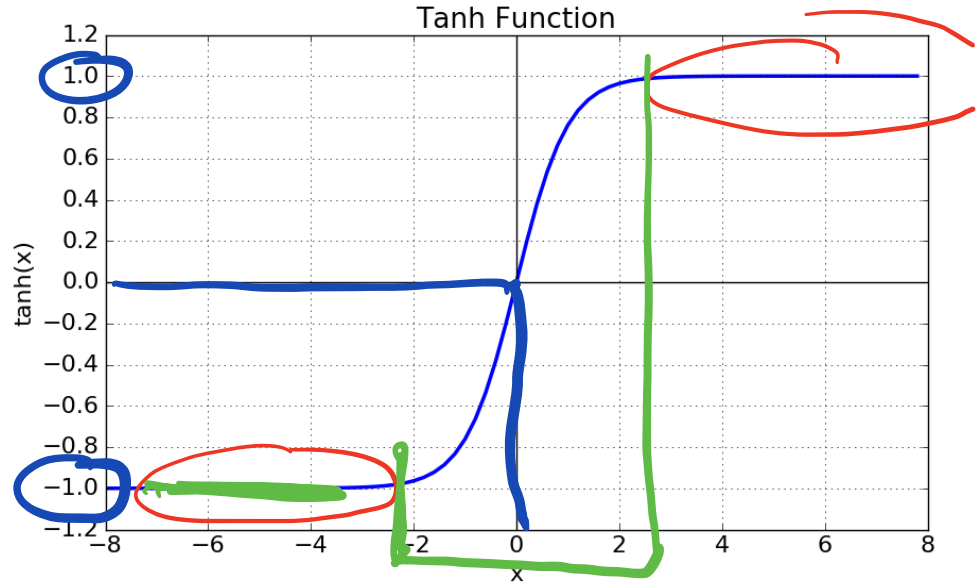
Use: Output layer for binary classification

Deep Learning

Activation Functions

- Tanh Function

$$\tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$



Range: [-1, 1]

Pros: Zero-centered (better than sigmoid), stronger gradients

Cons: Still has vanishing gradient problem

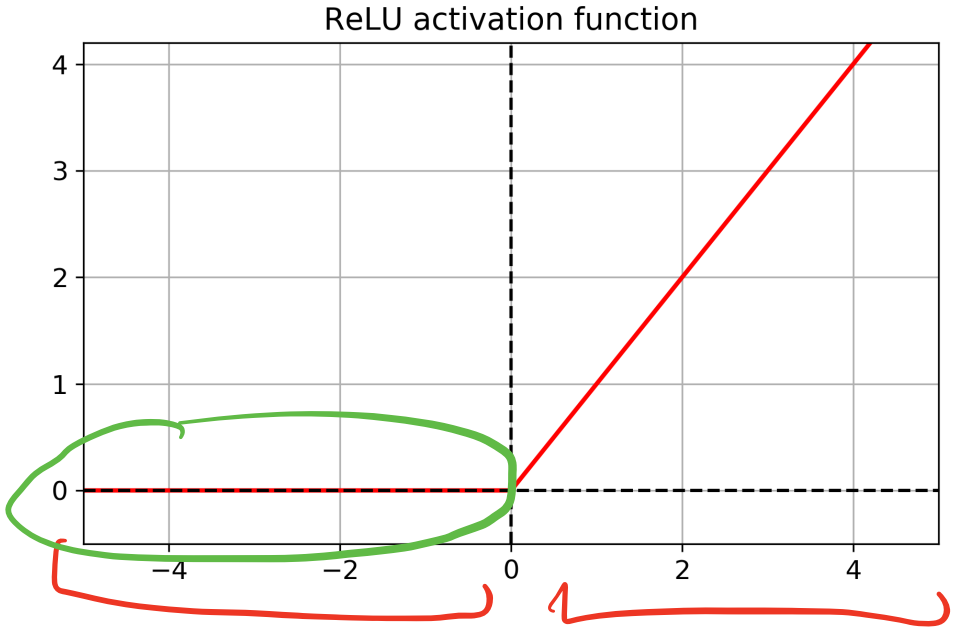
Use: Hidden layers (historically), RNNs

Deep Learning

Activation Functions

- ReLU Function (Rectified Linear Unit)

$$ReLU(x) = max(0, x) = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}$$



Range: [0, ∞)

Pros: No vanishing gradient (for x > 0), computationally efficient, sparse activation

Cons: “Dying ReLU” problem (neurons stuck at 0), not zero-centered

Use: Default for hidden layers in most networks

Deep Learning

Activation Functions

- Leaky ReLU Function

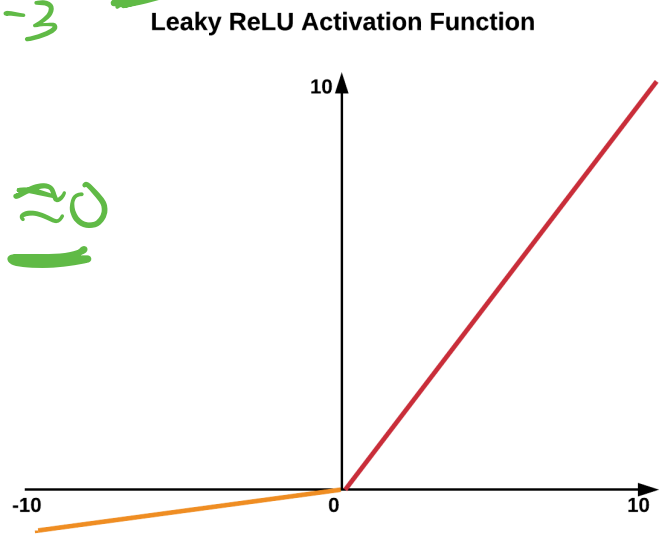
$$ReLU(x) = \begin{cases} x & x > 0 \\ \alpha x & x \leq 0 \end{cases}$$

Range: $(-\infty, \infty)$

Pros: Prevents dying ReLU

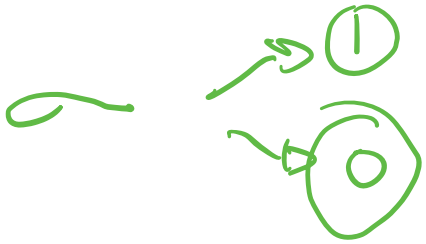
Cons: Extra hyperparameter $(\alpha = 0.01)$

Use: Default for hidden layers in most networks



Deep Learning

Activation Functions



- Softmax (**Output** Layer)

$$softmax(x)_k = \frac{e^{x_k}}{\sum_{j=1}^{K} e^{x_j}}$$

Output: Probability distribution over K classes (sums to 1)

Deep Learning

Activation Functions

- Softmax (**Output** Layer)

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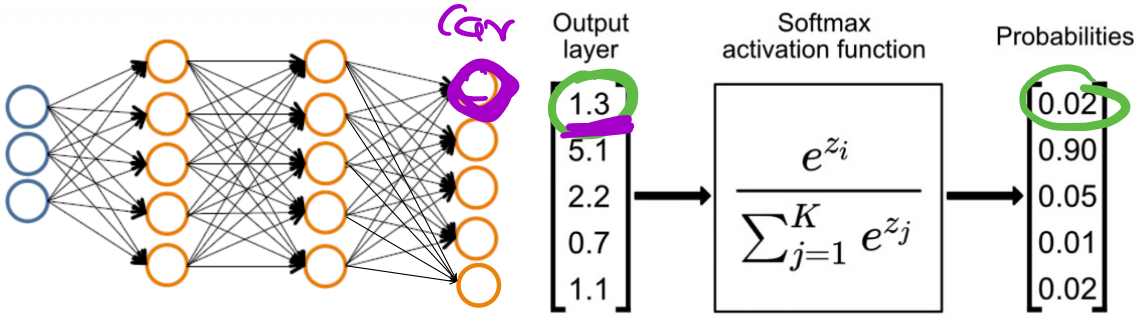


Deep Learning

Activation Functions

- Softmax (**Output** Layer)

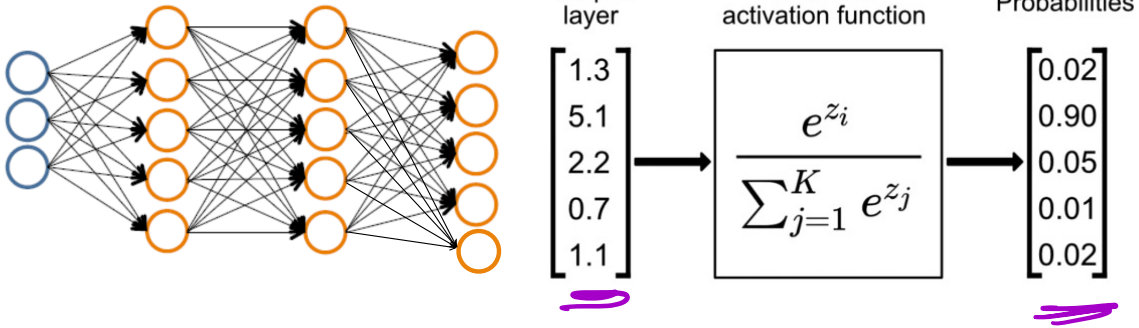
$$softmax(x)_k = \frac{e^{x_k}}{\sum_{j=1}^{K} e^{x_j}}$$

If we have multi-class probability, what loss function can we use?

Categorical Cross Entropy Loss:

$$\ell_\theta(x) = -\frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{K} y_{ik} log(\hat{y}_{ik})$$

Output: Probability distribution over K classes (sums to 1)



Deep Learning

Activation Functions

If we have multi-class probability, what loss function can we use?

Categorical Cross Entropy Loss:

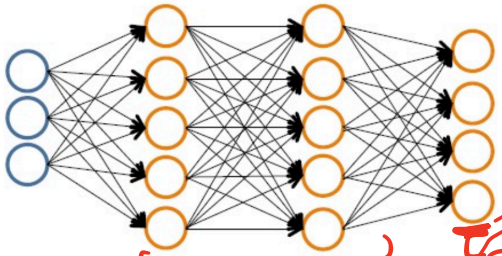
$$\ell_\theta(x) = -\frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{K} y_{ik} log(\hat{y}_{ik})$$


Pedestrian


Car


Motorcycle


Truck



$h_\Theta(\mathbf{x}) \in \mathbb{R}^K$

We want:

$h_\Theta(\mathbf{x}) \approx \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ when pedestrian

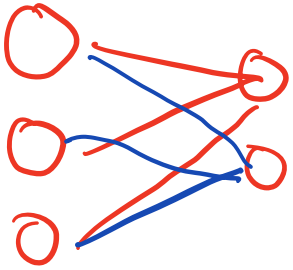
$h_\Theta(\mathbf{x}) \approx \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ when car

$h_\Theta(\mathbf{x}) \approx \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ when motorcycle

$h_\Theta(\mathbf{x}) \approx \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ when truck

Deep Learning

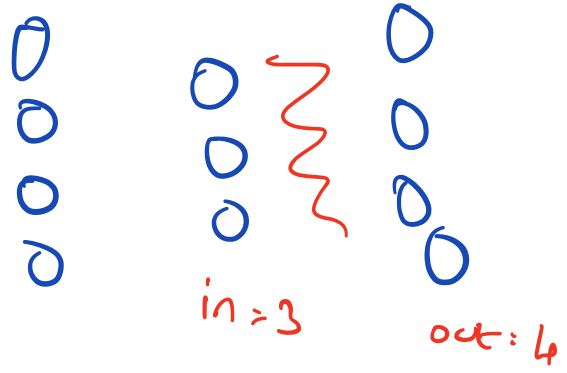
Weight Initialization



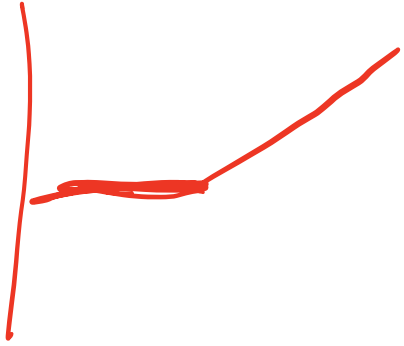
- Poor initialization causes:
  - **Vanishing signals**: Activations shrink to zero through layers
  - **Exploding signals**: Activations grow unboundedly
  - **Symmetry**: If all weights are equal, all neurons compute the same thing
    - **All Zeros**
      - Problem: All neurons compute the same thing. Gradients are identical. Symmetry is never broken. Network cannot learn.
    - **All Same Value**
      - Same problem as zeros - symmetry is not broken.

Deep Learning

Weight Initialization



- **Xavier/Glorot Initialization**
  - **Goal:** Keep variance of activations constant across layers.
  - Useful for sigmoid or tanh activations



$$W \sim \mathcal{N}(0, \frac{2}{n_{in} + n_{out}})$$

Normal or Gaussian

$$\mathcal{N}\left(0, \frac{2}{7}\right)$$

Deep Learning

Weight Initialization

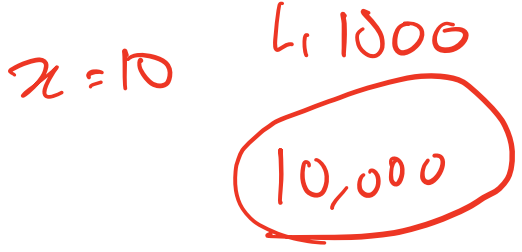
- **He Initialization**
  - **Goal:** Keep variance of activations constant across layers.
  - Useful for ReLU since ReLU zeros out half the activations (negative values), effectively halving the variance. Compensate by doubling

$$W \sim \mathcal{N}(0, \frac{2}{n_{in}})$$

Deep Learning

Regularization

- Deep networks have **millions** of parameters - they can **memorize** training data.
- We need regularization to improve generalization.



Deep Learning

Regularization

- L_2 Regularization
  - Add penalty on weight magnitude to loss

$$L_\theta(x) = L_\theta(x) + \lambda \sum_{l} \| W^{[l]} \|_F^2$$

  - **Effect**: Pushes weights toward zero, preferring simpler models.

Deep Learning
Regularization

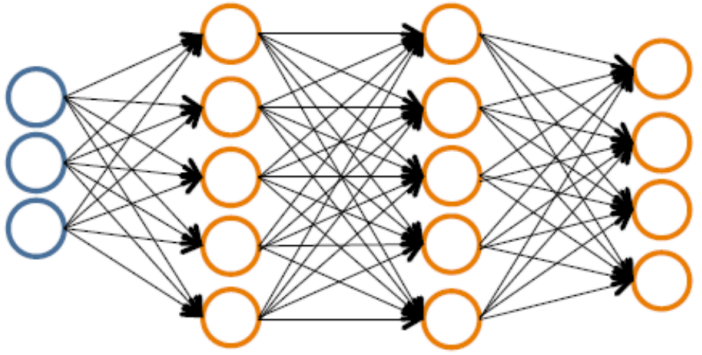
- L_1 Regularization
  - Add penalty on weight magnitude to loss

$$L_\theta(x) = L_\theta(x) + \lambda \sum_l \|W^{[l]}\|_1$$

  - **Effect**: Encourages **sparsity** (many weights become exactly zero)

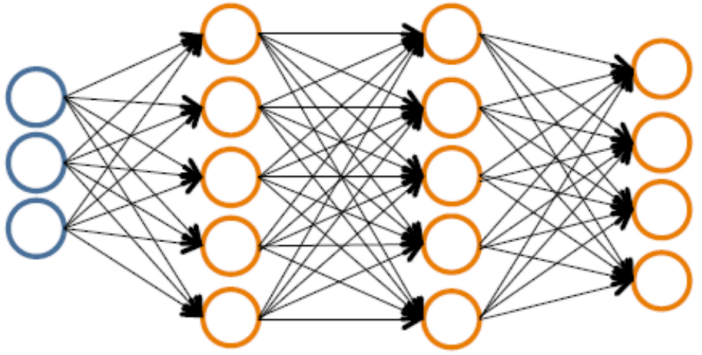
Deep Learning

Regularization



- Dropout Regularization
  - During training: randomly set activations to zero with probability p
  - During inference: Use all neurons, no dropout
  - **Intuition:**
    - Forces network to not rely on any single neuron
    - Approximately trains an ensemble of 2^n sub-networks
    - Prevents **co-adaptation** of neurons

Deep Learning
Regularization



- Batch Normalization
  - Normalize **activations** within each mini-batch

$$\hat{z} = \frac{z - \mu_B}{\sqrt{\sigma_B^2}}$$

$$\tilde{z} = \gamma \cdot \hat{z} + \beta$$

  - **Benefits**:
    - Reduces internal covariate shift
    - Allows higher learning rates
    - Acts as regularization (noise from batch statistics)
    - Reduces sensitivity to initialization

Deep Learning

Hyperparameters

- Number of layers
- Number of hidden units (neurons) per layer
- Activation function
- Initialization
- Regularization

Deep Learning

Backpropagation

- Compute gradients for each weight using the chain rule
- Consider a 2-layer MLP with the following forward pass:

Layer 1 (Hidden Layer):

$$z^{[1]} = W^{[1]}x + b^{[1]}$$
$$a^{[1]} = \text{ReLU}(z^{[1]})$$

Layer 2 (Output Layer):

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$
$$\hat{y} = \sigma(z^{[2]})$$

Deep Learning

Backpropagation

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$$a^{[1]} = \text{ReLU}(z^{[1]})$$

Layer 2 (Output Layer):

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$
$$\hat{y} = \sigma(z^{[2]})$$

Loss Function: $L = -\left[y \log \hat{y} + (1-y)\log(1-\hat{y})\right]$

Forward Pass: Compute and store all intermediate values:

$$x \xrightarrow{W^{[1]},b^{[1]}} z^{[1]} \xrightarrow{\text{ReLU}} a^{[1]} \xrightarrow{W^{[2]},b^{[2]}} z^{[2]} \xrightarrow{\sigma} \hat{y} \to L$$

Deep Learning

Backpropagation

Forward Pass: Compute and store all intermediate values:

$$x \xrightarrow{W^{[1]},b^{[1]}} z^{[1]} \xrightarrow{\text{ReLU}} a^{[1]} \xrightarrow{W^{[2]},b^{[2]}} z^{[2]} \xrightarrow{\sigma} \hat{y} \to L$$

Backward Pass: We compute gradients layer by layer, from output to input, using the chain rule.

We need $\frac{\partial L}{\partial z^{[2]}}$. Using the chain rule:

$$\frac{\partial L}{\partial z^{[2]}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z^{[2]}}$$

or binary cross-entropy loss:

$$\frac{\partial L}{\partial \hat{y}} = -\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}}$$

Deep Learning

Backpropagation

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or binary cross-entropy loss:

$$\frac{\partial L}{\partial \hat{y}} = -\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}}$$

Combining these (with simplification):

$$\delta^{[2]} = \frac{\partial L}{\partial z^{[2]}} = \hat{y} - y$$

Deep Learning

Backpropagation

Backward Pass: We compute gradients layer by layer, from output to input, using the chain rule.

Step 2: Gradients for $W^{[2]}$ and $b^{[2]}$

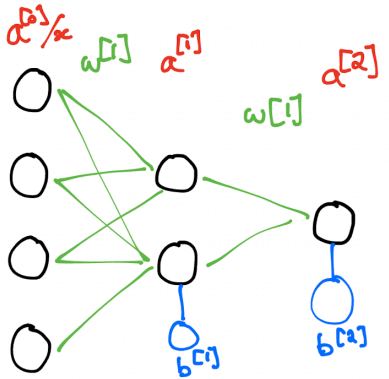
Since $z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$, we have:

Weight gradient: $$\frac{\partial L}{\partial W^{[2]}} = \frac{\partial L}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial W^{[2]}} = \delta^{[2]} \cdot \frac{\partial}{\partial W^{[2]}} \left(W^{[2]}a^{[1]} + b^{[2]} \right)$$

$$\frac{\partial L}{\partial W^{[2]}} = \delta^{[2]} (a^{[1]})^T$$

Deep Learning

Backpropagation



Equations –

$$z^{[1]} = w^{[1]}a^{[0]} + b^{[1]}$$

$(2\times 1)\times(1\times m)$

$$a^{[1]} = \sigma(z^{[1]})$$

$(2\times m)$

$$z^{[2]} = w^{[2]}a^{[1]} + b^{[2]}$$

$(1\times 2)\times(2\times m)$

$$a^{[2]} = \sigma(z^{[2]})$$

$(1\times m)$

$$\mathcal{L} = \frac{1}{2}(y-\hat{y})^2$$

$$\frac{\partial \mathcal{L}}{\partial a^{[2]}} = \frac{\partial}{\partial a^{[2]}}\left(\frac{1}{2}(y-\hat{y})^2\right)$$

$$= \frac{\partial}{\partial a^{[2]}}\left(\frac{1}{2}(y-a^{[2]})^2\right)$$

$$= -(y-a^{[2]})$$

$$\frac{\partial a^{[2]}}{\partial z^{[2]}} = \frac{\partial}{\partial z^{[2]}}\left(\sigma(z^{[2]})\right)$$

$$= \sigma(z^{[2]})(1-\sigma(z^{[2]}))$$

$$\frac{\partial z^{[2]}}{\partial a^{[1]}} = \frac{\partial}{\partial a^{[1]}}\left(w^{[2]}a^{[1]} + b^{[2]}\right)$$

$$= w^{[2]}$$

$$\frac{\partial z^{[2]}}{\partial w^{[2]}} = \frac{\partial}{\partial w^{[2]}}\left(w^{[2]}a^{[1]} + b^{[2]}\right)$$

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$$\frac{\partial z^{[1]}}{\partial a^{[0]}} = \frac{\partial}{\partial a^{[0]}}\left(w^{[1]}a^{[0]} + b^{[1]}\right)$$

$$= w^{[1]}$$

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$$= a^{[0]}$$

Chain Rule –

$$da^{[2]} = \frac{\partial \mathcal{L}}{\partial a^{[2]}} = -(y-a^{[2]}) = a^{[2]} - y$$

$$dz^{[2]} = \frac{\partial \mathcal{L}}{\partial z^{[2]}} = \left(\frac{\partial \mathcal{L}}{\partial a^{[2]}}\right)\left(\frac{\partial a^{[2]}}{\partial z^{[2]}}\right)$$

$$= da^{[2]}\, g'(z^{[2]})$$

$$dw^{[2]} = \frac{\partial \mathcal{L}}{\partial w^{[2]}} = \left(\frac{\partial \mathcal{L}}{\partial a^{[2]}}\right)\left(\frac{\partial a^{[2]}}{\partial z^{[2]}}\right)\left(\frac{\partial z^{[2]}}{\partial w^{[2]}}\right)$$

$$= \underbrace{-(y-a^{[2]})(\sigma(z^{[2]})(1-\sigma(z^{[2]})))}_{dz^{[2]}}\, a^{[1]}$$

$$da^{[1]} = \frac{\partial \mathcal{L}}{\partial a^{[1]}} = \underbrace{\left(\frac{\partial \mathcal{L}}{\partial a^{[2]}}\right)\left(\frac{\partial a^{[2]}}{\partial z^{[2]}}\right)}_{dz^{[2]}}\left(\frac{\partial z^{[2]}}{\partial a^{[1]}}\right)$$

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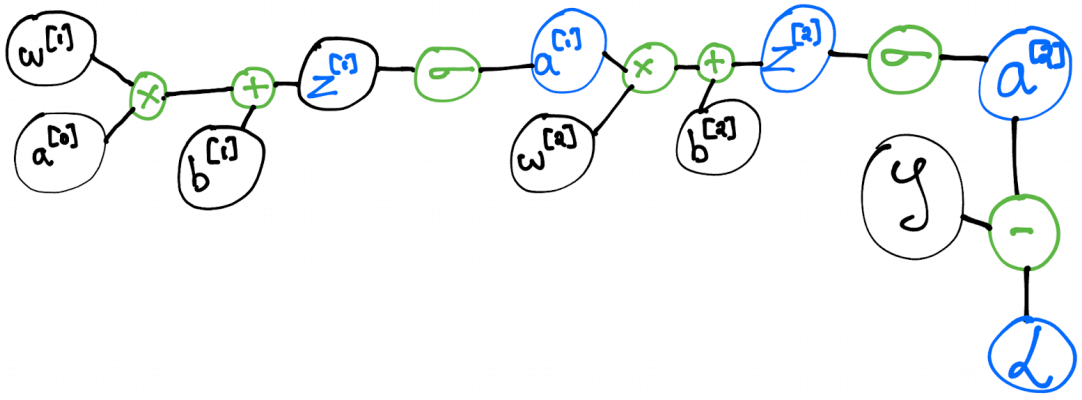
Deep Learning

Backpropagation



Final Equations-

$$da^{[2]} = a^{[2]} - y$$
$$dz^{[2]} = da^{[2]}\, g'(z^{[2]})$$
$$dw^{[2]} = dz^{[2]}\, a^{[1]}$$
$$da^{[1]} = dz^{[2]}\, w^{[2]}$$
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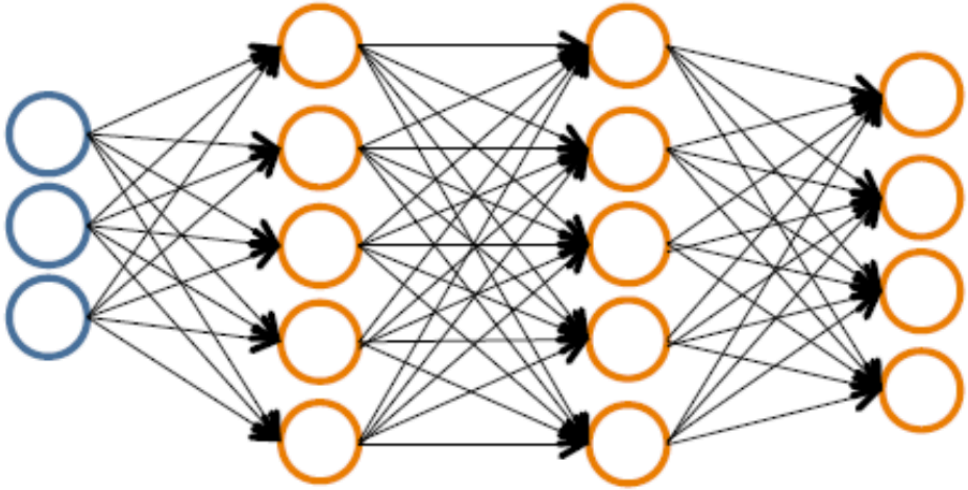
Deep Learning

Backpropagation

1. Forward Propagation



2. Backward Propagation

3. ?

Equations –

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$(2 \times 4) \times (4 \times m)$

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Deep Learning

Backpropagation

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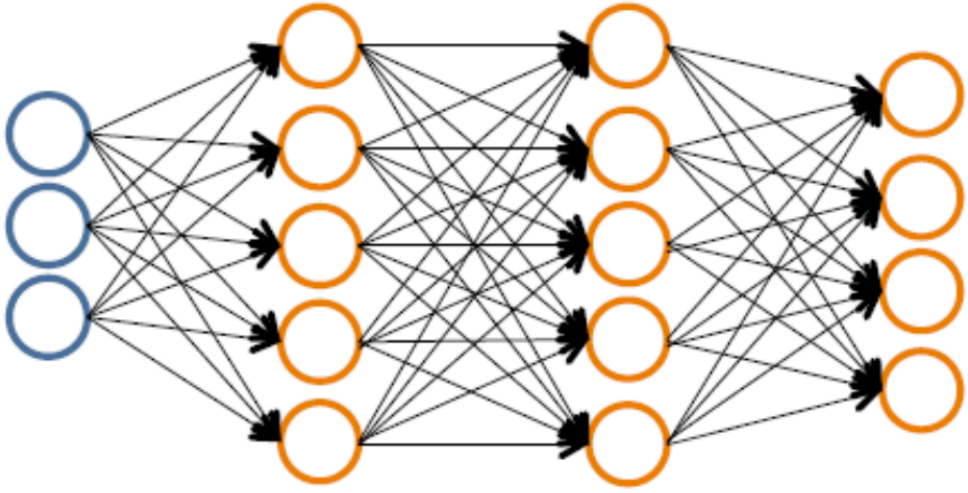

2. Backward Propagation

3. Update Weights - Gradient Descent

Equations –

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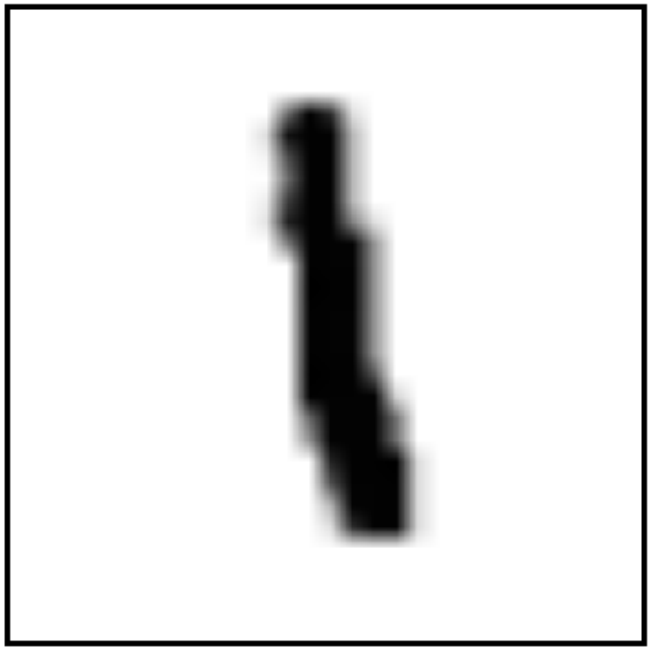
Today’s Outline

- Deep Learning
- Convolutional Neural Networks

Convolutional Neural Networks

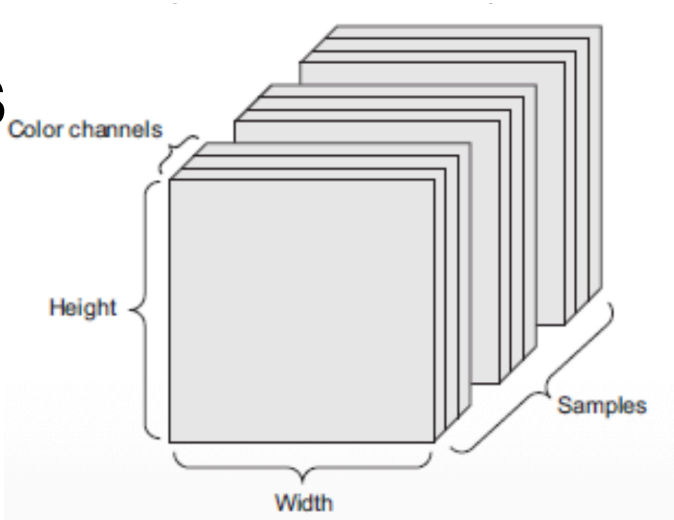
- An image is represented as a grid of pixel values



$\approx$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .6 & .8 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .7 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .7 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .5 & 1 & .4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & .4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & .4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & .7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & .9 & 1 & .1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & .3 & 1 & .1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Convolutional Neural Networks



- An image is represented as a grid of pixel values
- Lets say this image os 224x224 RGB image
  - Input size after flattening out image = 224 x 224 x 3 = 150,528
  - Input of neural network needs 150,528 neurons
  - Assume first hidden layer has 1000 neurons, first weight matrix will have 150,528 x 1000 =150,528,000 ~ 150 million parameters in the first layer alone.

Convolutional Neural Networks

- An image is represented as a grid of pixel values
- Lets say this image os 224x224 RGB image
  - ~ 150 million parameters in the first layer alone
  - Why is this a problem?
    - **Too many parameters**: Overfitting, memory issues
    - **No spatial structure**: Flattening destroys 2D relationships
    - **No translation invariance**: A cat in the corner vs. center looks completely different to an MLP

Convolutional Neural Networks

- Key insights from human vision
  - **Local connectivity:** Pixels that are close together are more related than distant pixels.
    - A neuron doesn't need to see the entire image.

Convolutional Neural Networks

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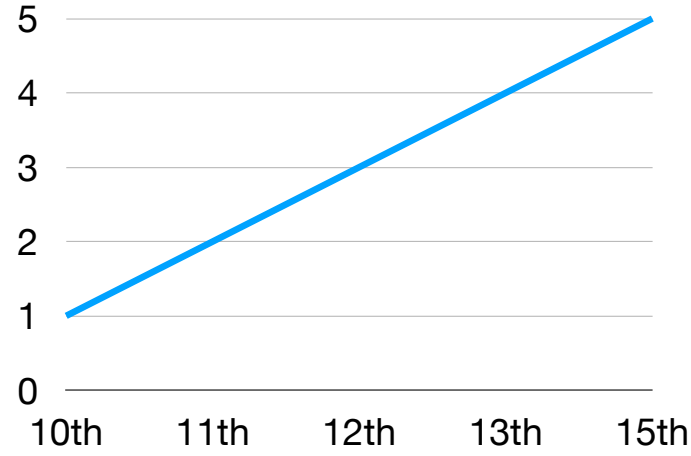
  - **Local connectivity:** Pixels that are close together are more related than distant pixels.
    - A neuron doesn't need to see the entire image.
  - **Translation invariance:** A feature (like an edge or eye) should be detected regardless of its position in the image.
  - **Hierarchical structure:** Simple features (edges) combine to form complex features (shapes → objects).

Convolutional Neural Networks

- CNNs exploit these insights through:
  - **Local receptive fields**: Each neuron sees only a small region
  - **Weight sharing:** Same detector (filter) applied across entire image
  - **Pooling:** Reduces spatial dimensions, adds invariance

Convolutional Neural Networks

- Understanding a 1D Filter:
  - Signal: $[1 \quad 2 \quad 3 \quad 4 \quad 5]$
  - Filter: $[1 \quad 0 \quad -1]$



Convolutional Neural Networks

- Understanding a 1D Filter:
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  - Position 0: $(1 \cdot 1) + (2 \cdot 0) + (2 \cdot -1) = -2$
  - Position 1: $(2 \cdot 1) + (3 \cdot 0) + (4 \cdot -1) = -2$
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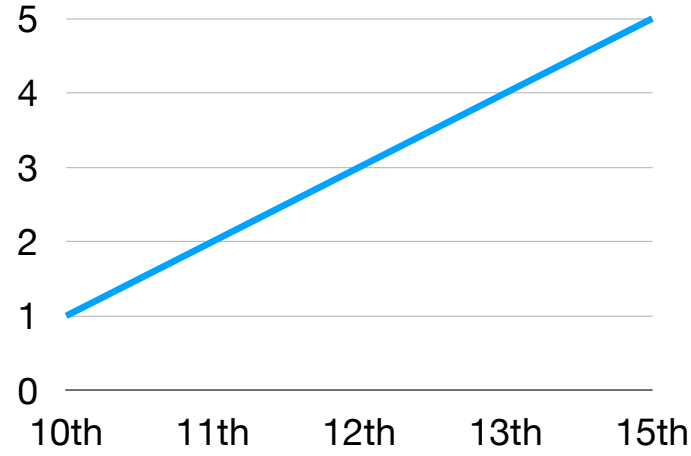
  - Output: $[-2 \quad -2 \quad -2]$



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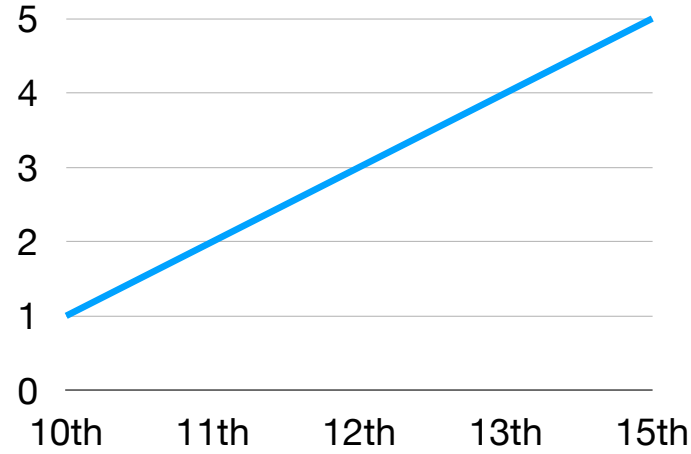

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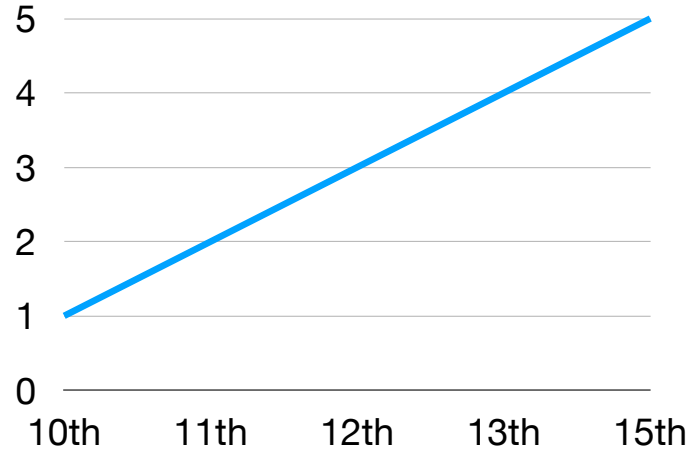

Convolutional Neural Networks

- These filters “slide” over the entire dataset
- Each filter learns something about the data
- 2D filters similarly slide over an image

Convolutional Neural Networks

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$ — Vertical Edge Filter

$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \end{bmatrix}$$

Convolutional Neural Networks

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Convolutional Neural Networks

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Convolutional Neural Networks

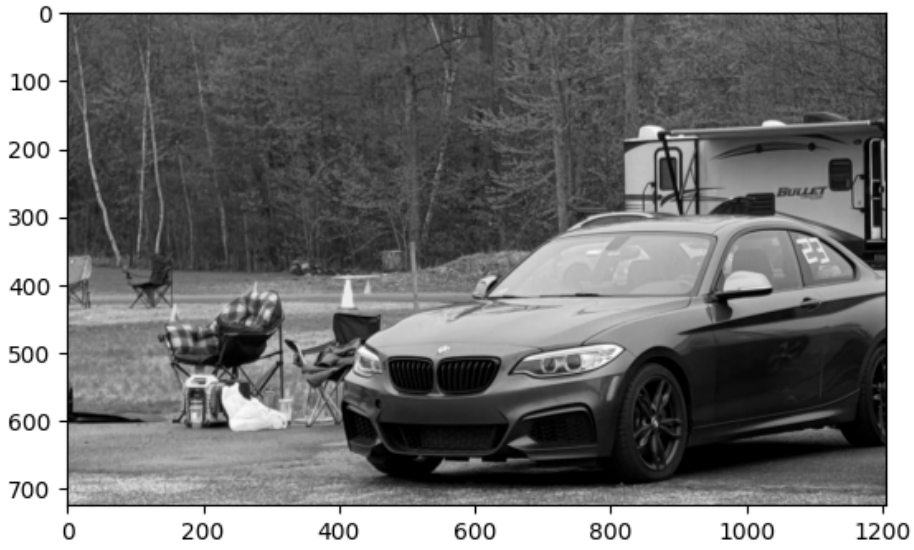
$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$ — Vertical Edge Filter

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$ — Blur (Averaging)

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$ — Vertical Edge Filter

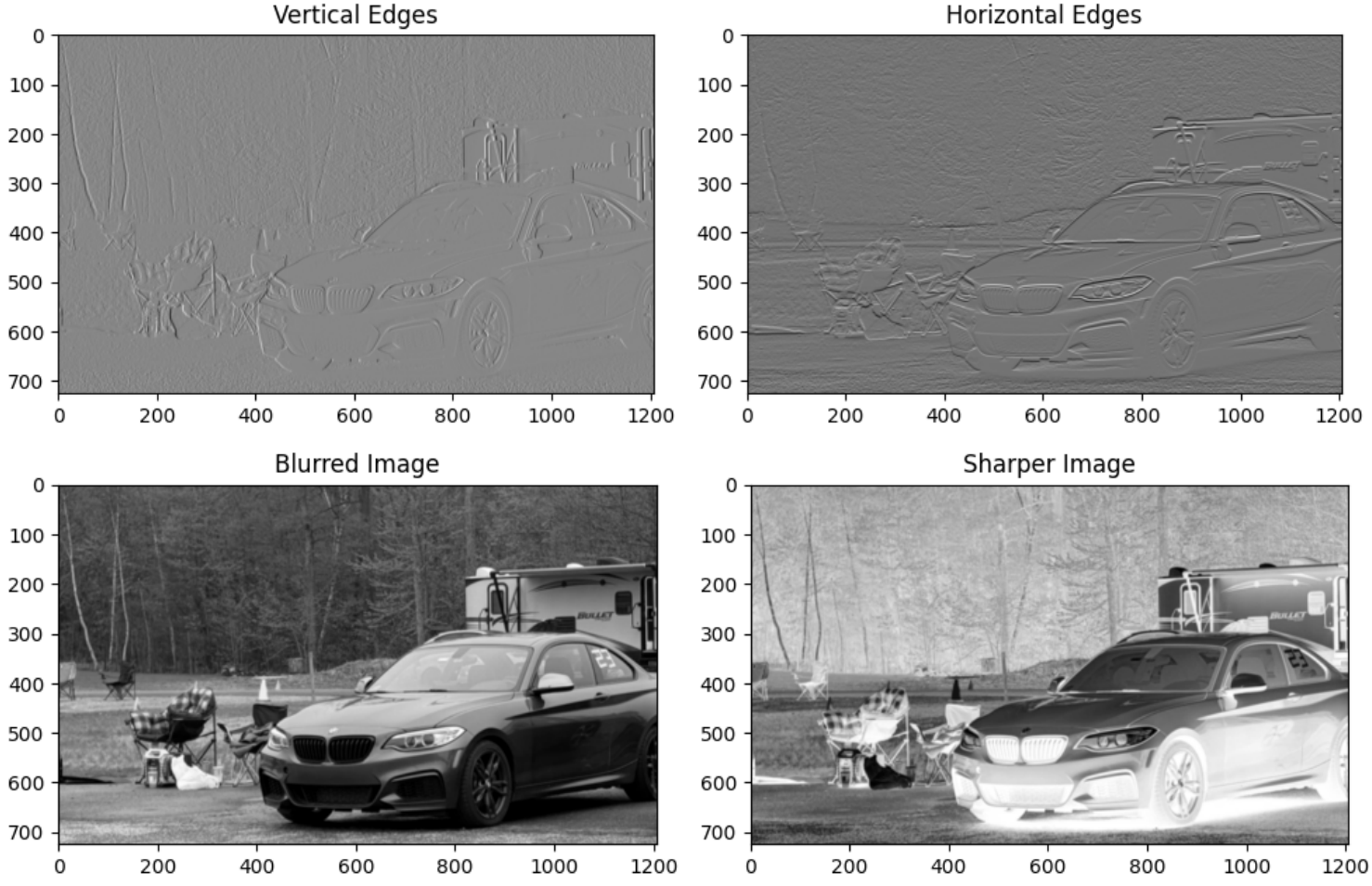
$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$ — Sharpen

Convolutional Neural Networks



Original

Convolutional Neural Networks



Convolutional Neural Networks

- Key idea of CNNs - **learn filter values** rather than hand designing them
- A convolutional layer has **multiple filters**, each producing one output channel (feature map).

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- You learn filters like

$$\begin{bmatrix} \theta_{10} & \theta_{11} & \theta_{12} \\ \theta_{13} & \theta_{14} & \theta_{15} \\ \theta_{16} & \theta_{17} & \theta_{18} \end{bmatrix}$$

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$$\text{Filter 1: } \begin{bmatrix} \theta_{10} & \theta_{11} & \theta_{12} \\ \theta_{13} & \theta_{14} & \theta_{15} \\ \theta_{16} & \theta_{17} & \theta_{18} \end{bmatrix} \quad \text{Filter 2: } \begin{bmatrix} \theta_{20} & \theta_{21} & \theta_{22} \\ \theta_{23} & \theta_{24} & \theta_{25} \\ \theta_{26} & \theta_{27} & \theta_{28} \end{bmatrix}$$

Convolutional Neural Networks

- Key idea of CNNs - **learn filter value**s rather than hand designing them
- A convolutional layer has **multiple filters**, each producing one output channel (feature map).
- Input: $H_{in} \times W_{in} \times C_{in}$ (height x width x channels)
- Output: $H_{out} \times W_{out} \times C_{out}$ (one channel per filter)
- Each filter has shape $k \times k \times C_{in}$

Convolutional Neural Networks

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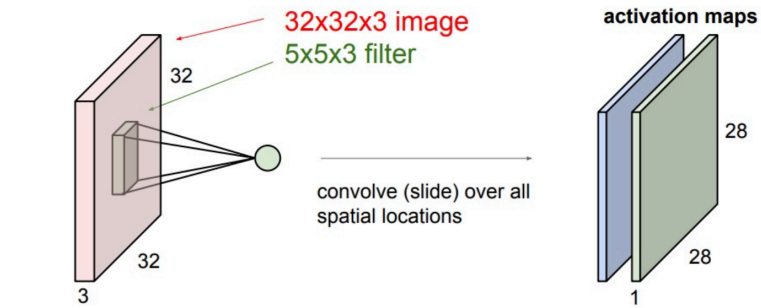
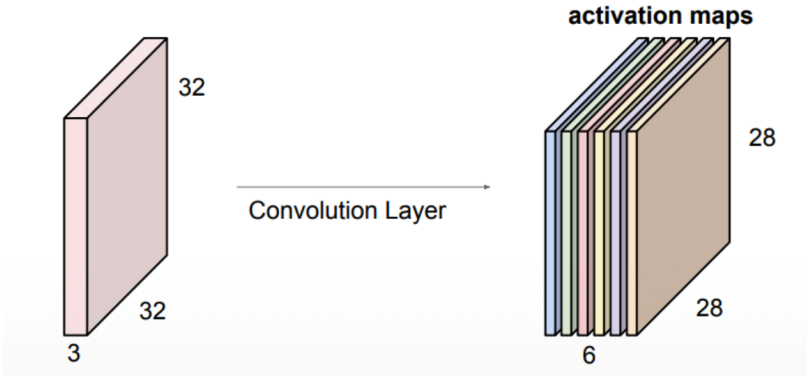
- Output: $H_{out} \times W_{out} \times C_{out}$ (one channel per filter)
- Each filter has shape $k \times k \times C_{in}$
- **Example**
  - Input Image: $32 \times 32 \times 3$ (RGB Image), 16 filters
  - Each filter: $5 \times 5 \times 3 = 75$ parameters
  - Output Shape: $28 \times 28 \times 16$ (why 28?)
- Total parameters = 75 x 16 = 1200, compared to 32 x 32 x 3 x 1000 = 3M parameters for MLP





Convolutional Neural Networks

Stride

- Output Shape: $28 \times 28 \times 16$ (why 28?)
- Stride captures how many pixels the filter moves each step

$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & -30 & 0 \\ 0 & 30 & -30 & 0 \\ 0 & 30 & -30 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & -30 & 0 \\ 0 & 30 & -30 & 0 \\ 0 & 30 & -30 & 0 \end{bmatrix}$$

Convolutional Neural Networks

Stride

- Output Shape: $28 \times 28 \times 16$ (why 28?)
- Stride captures how many pixels the filter moves each step

Stride = 1

$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & -30 & 0 \\ 0 & 30 & -30 & 0 \\ 0 & 30 & -30 & 0 \end{bmatrix}$$

Convolutional Neural Networks

Stride and Padding

- Output Shape: $28 \times 28 \times 16$ (why 28?)
- Stride captures how many pixels the filter moves each step

Stride = 2

$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & -30 & 0 \\ 0 & 30 & -30 & 0 \\ 0 & 30 & -30 & 0 \end{bmatrix}$$

Convolutional Neural Networks

Stride and Padding

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Convolutional Neural Networks

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Stride = 2

Next Stride?

Convolutional Neural Networks

Stride and Padding

- Output Shape: $28 \times 28 \times 16$ (why 28?)
- Stride captures how many pixels the filter moves each step

$$\begin{bmatrix} 0 & 10 & 10 & 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 10 & 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & -30 & 0 \\ 0 & 30 & -30 & 0 \\ 0 & 30 & -30 & 0 \end{bmatrix}$$

Stride = 2

Padding

Convolutional Neural Networks

Stride and Padding

- Output Shape: $28 \times 28 \times 16$ (why 28?)
- Stride captures how many pixels the filter moves each step
- Stride and padding change the shape of the output matrix
- With no padding, input size n, filter size k and stride s :
  - Output Size = $\lfloor \frac{n-k}{s} \rfloor + 1$
- With padding p:
  - Output Size = $\lfloor \frac{n+2p-k}{s} \rfloor + 1$

Convolutional Neural Networks

Stride and Padding

$$\text{Output Size} = \left\lfloor \frac{n + 2p - k}{s} \right\rfloor + 1$$

Input (s)	Filter (k)	Padding (p)	Stride (s)	Output
32x32	3	0	1	?
32x32	3	1	1	?
32x32	3	2	2	?

Convolutional Neural Networks

Stride and Padding

$$\text{Output Size} = \left\lfloor \frac{n + 2p - k}{s} \right\rfloor + 1$$

Input (s)	Filter (k)	Padding (p)	Stride (s)	Output
32x32	3	0	1	30
32x32	3	1	1	32
32x32	3	2	2	17

Convolutional Neural Networks

Stride and Padding

Output Size = $\lfloor \frac{n + 2p - k}{s} \rfloor + 1$

Input (s)	Filter (k)	Padding (p)	Stride (s)	Output
32x32	3	0	1	30
32x32	3	1	1	32
32x32	3	2	2	17

To keep same output size, pad by $\lfloor \frac{k}{2} \rfloor$ on each side

Convolutional Neural Networks

Pooling

- Pooling reduces spatial dimensions, providing:
  - Translation invariance
  - Reduced computation
  - Larger receptive field
- Example - **Max** Pool = (3x3) with stride = 3

$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 10 & 0 \\ 10 & 0 \end{bmatrix}$$

Convolutional Neural Networks

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Convolutional Neural Networks

Pooling

- Pooling reduces spatial dimensions, providing:
  - Translation invariance
  - Reduced computation
  - Larger receptive field
- Example - **Average** Pool = (3x3) with stride = 3

$$\begin{bmatrix} 1 & 10 & 1 & 0 & 1 & 0 \\ 1 & 10 & 1 & 0 & 1 & 0 \\ 1 & 10 & 1 & 0 & 1 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 0.3 \\ 3.3 & 0.3 \end{bmatrix}$$

Convolutional Neural Networks

Pooling

- Pooling reduces spatial dimensions, providing:
  - Translation invariance
  - Reduced computation
  - Larger receptive field
- Example - **Average** Pool = (3x3) with stride = 3

$$\begin{bmatrix} 1 & 10 & 1 & 0 & 1 & 0 \\ 1 & 10 & 1 & 0 & 1 & 0 \\ 1 & 10 & 1 & 0 & 1 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 0.3 \\ 3.3 & 0.3 \end{bmatrix}$$

Convolutional Neural Networks

Pooling

- Pooling reduces spatial dimensions, providing:
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  - Reduced computation
  - Larger receptive field
- Example - **Average** Pool = (3x3) with stride = 3

$$\begin{bmatrix} 1 & 10 & 1 & 0 & 1 & 0 \\ 1 & 10 & 1 & 0 & 1 & 0 \\ 1 & 10 & 1 & 0 & 1 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 0.3 \\ 3.3 & 0.3 \end{bmatrix}$$

Convolutional Neural Networks

Pooling

- Pooling reduces spatial dimensions, providing:
  - Translation invariance
  - Reduced computation
  - **Larger receptive field**
- Example - **Average** Pool = (3x3) with stride = 3

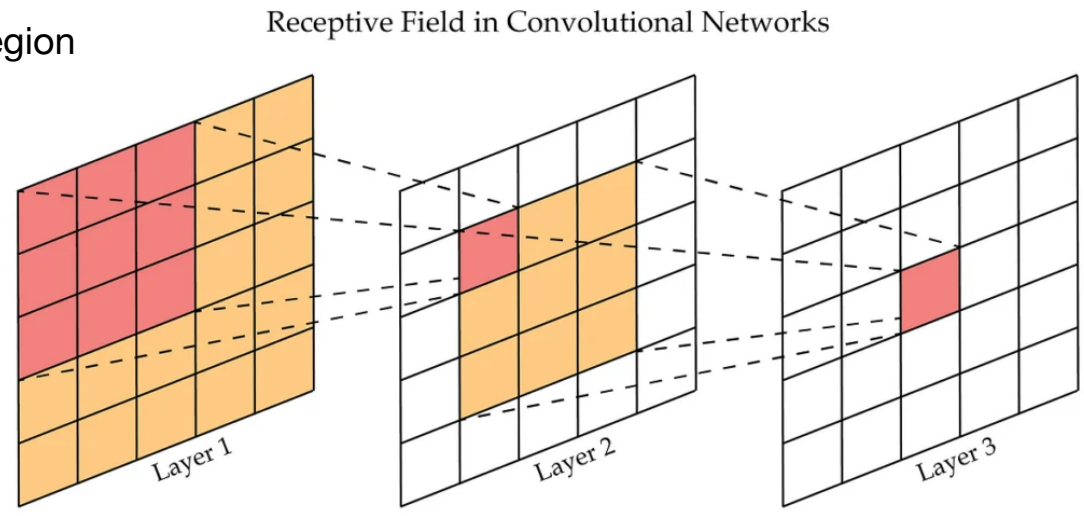
$$\begin{bmatrix} 1 & 10 & 1 & 0 & 1 & 0 \\ 1 & 10 & 1 & 0 & 1 & 0 \\ 1 & 10 & 1 & 0 & 1 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 0.3 \\ 3.3 & 0.3 \end{bmatrix}$$

Convolutional Neural Networks

Pooling

- The receptive field is the **region of the input** that influences a particular output neuron.
- With stacking, receptive fields grow:
  - Layer 1 (3×3 filter): Each output sees 3×3 input region
  - Layer 2 (3×3 filter): Each output sees 5×5 input region
  - Layer 3 (3×3 filter): Each output sees 7×7 input region

$$\begin{bmatrix} 1 & 10 & 1 & 0 & 1 & 0 \\ 1 & 10 & 1 & 0 & 1 & 0 \\ 1 & 10 & 1 & 0 & 1 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 0.3 \\ 3.3 & 0.3 \end{bmatrix}$$



Convolutional Neural Networks

Pooling

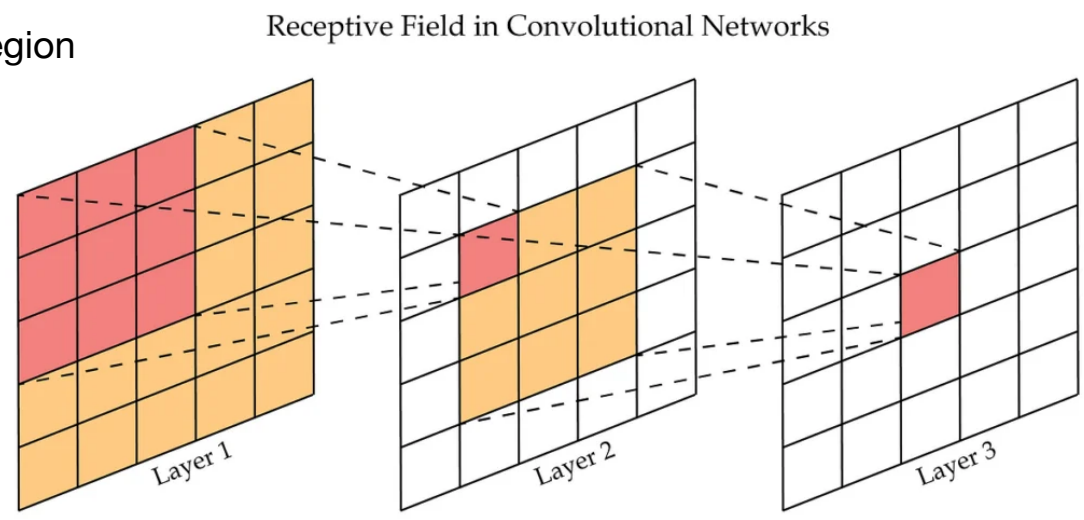
Why it matters: Deeper layers have larger receptive fields, allowing them to detect larger, more complex patterns.

- The receptive field is the **region of the input** that influences a particular output neuron.
- With stacking, receptive fields grow:
  - Layer 1 (3×3 filter): Each output sees 3×3 input region
  - Layer 2 (3×3 filter): Each output sees 5×5 input region
  - Layer 3 (3×3 filter): Each output sees 7×7 input region

Formula for L layers of $k \times k$ filters with stride 1:

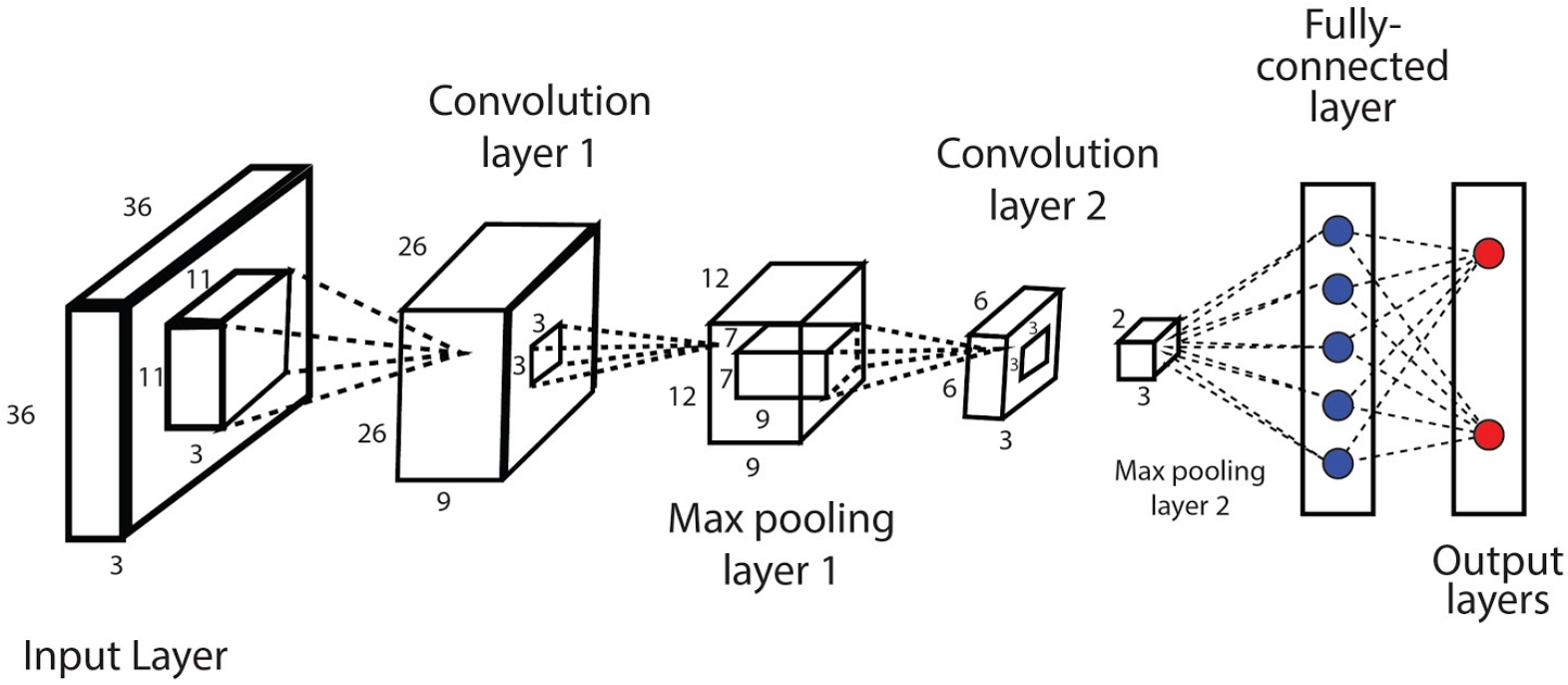
$$RF = L \times (k - 1) + 1$$

$$\begin{bmatrix} 1 & 10 & 1 & 0 & 1 & 0 \\ 1 & 10 & 1 & 0 & 1 & 0 \\ 1 & 10 & 1 & 0 & 1 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 0.3 \\ 3.3 & 0.3 \end{bmatrix}$$



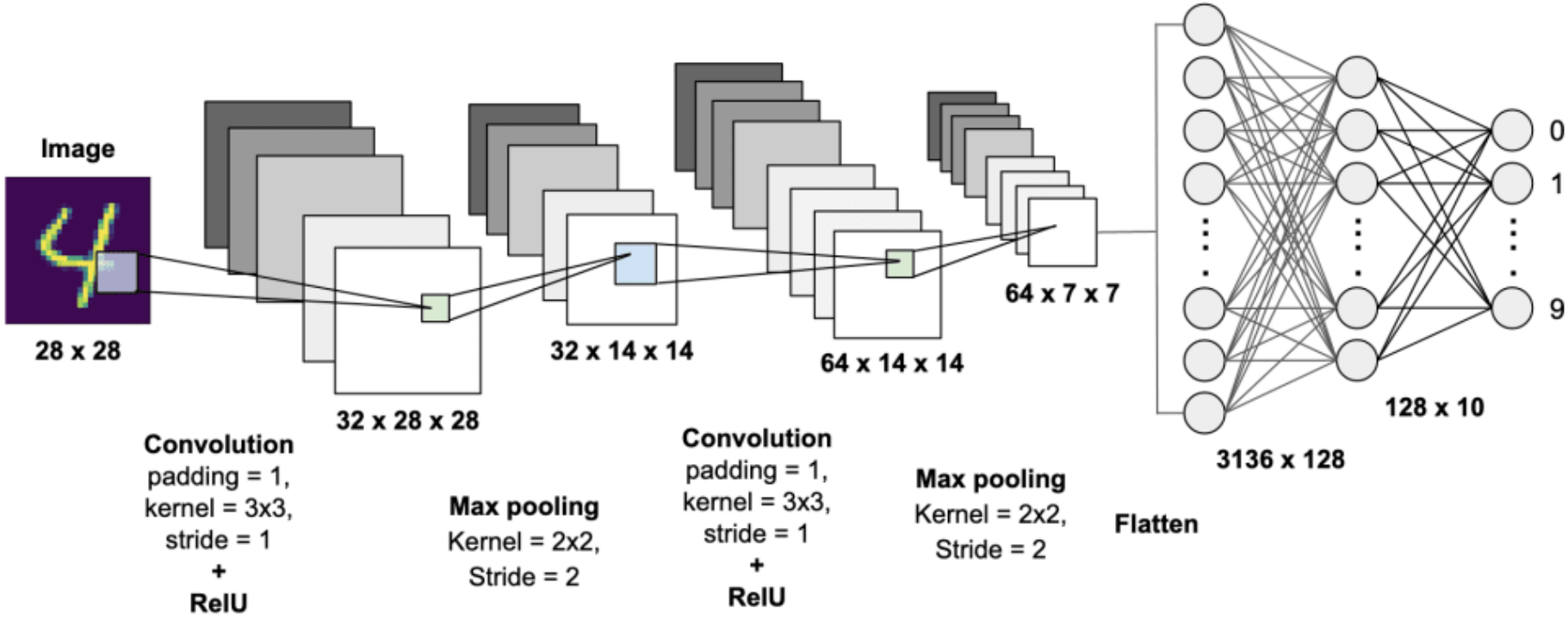
Convolutional Neural Networks

Overall Architecture



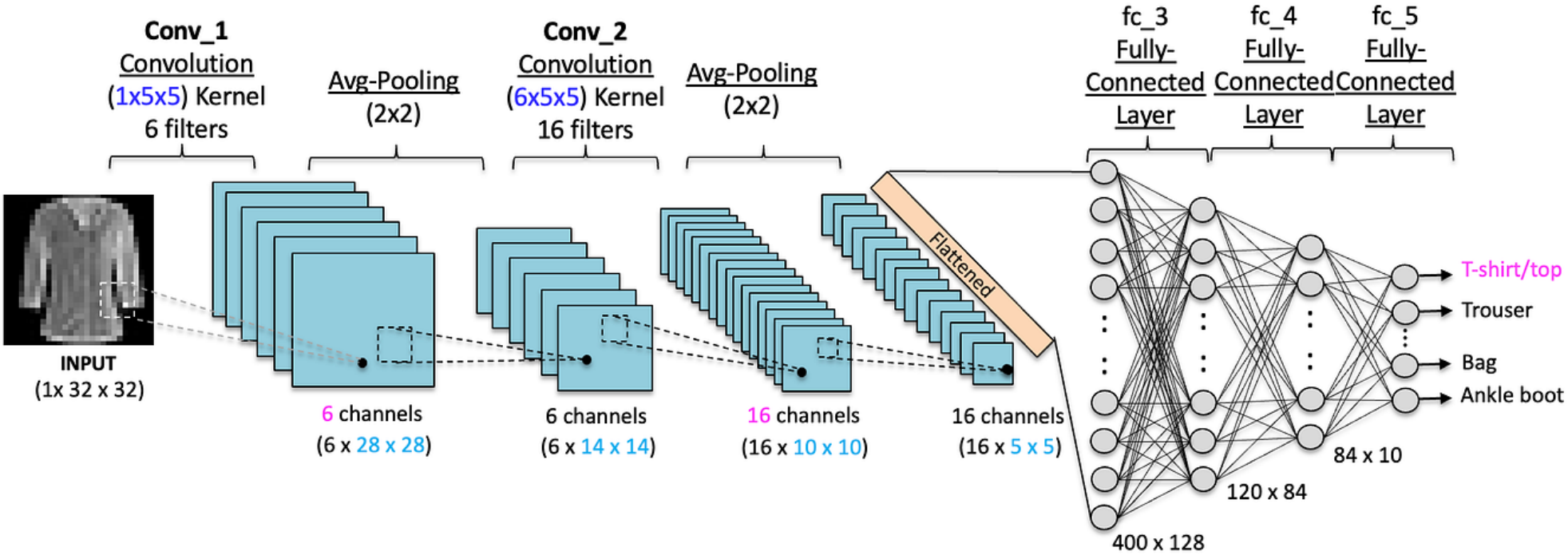
Convolutional Neural Networks

Overall Architecture



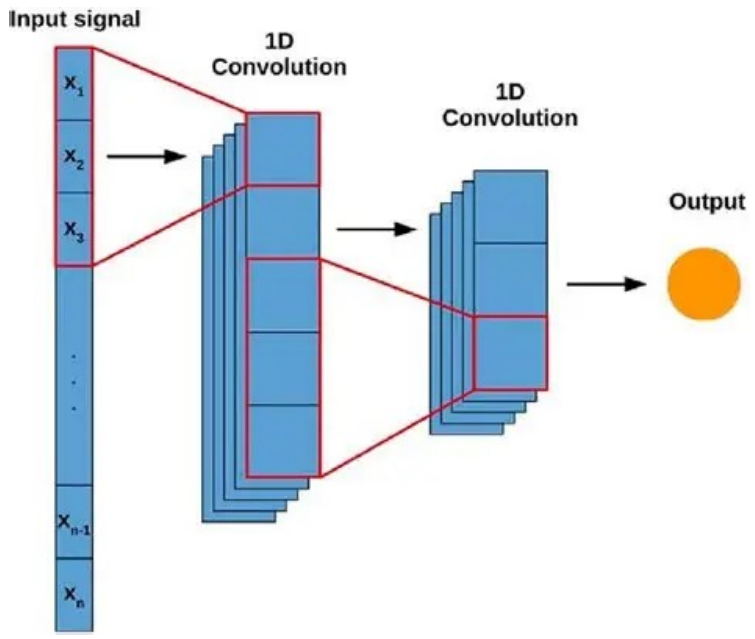
Convolutional Neural Networks

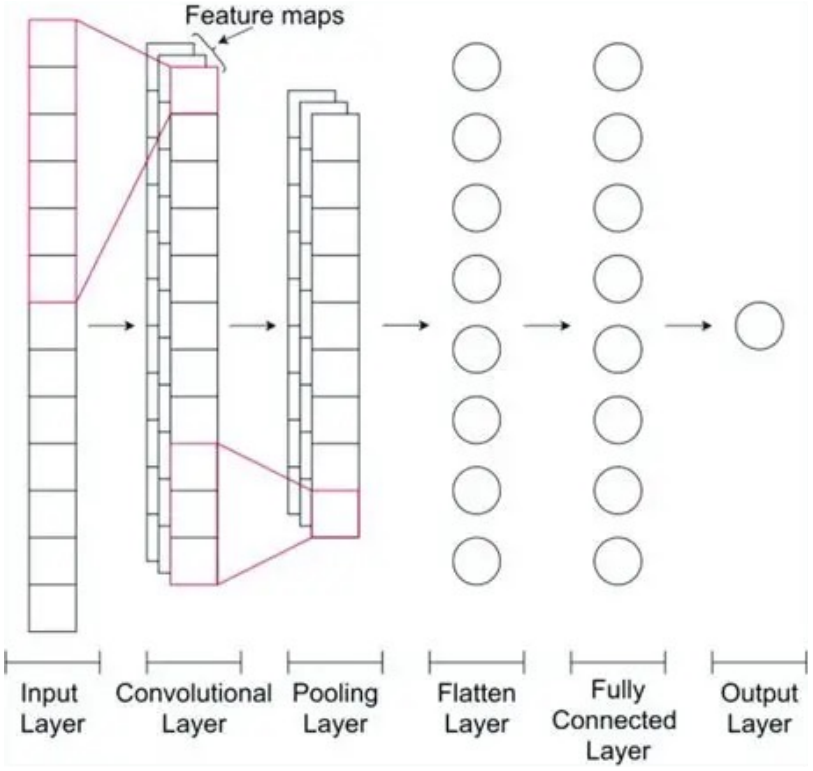
Overall Architecture



Convolutional Neural Networks

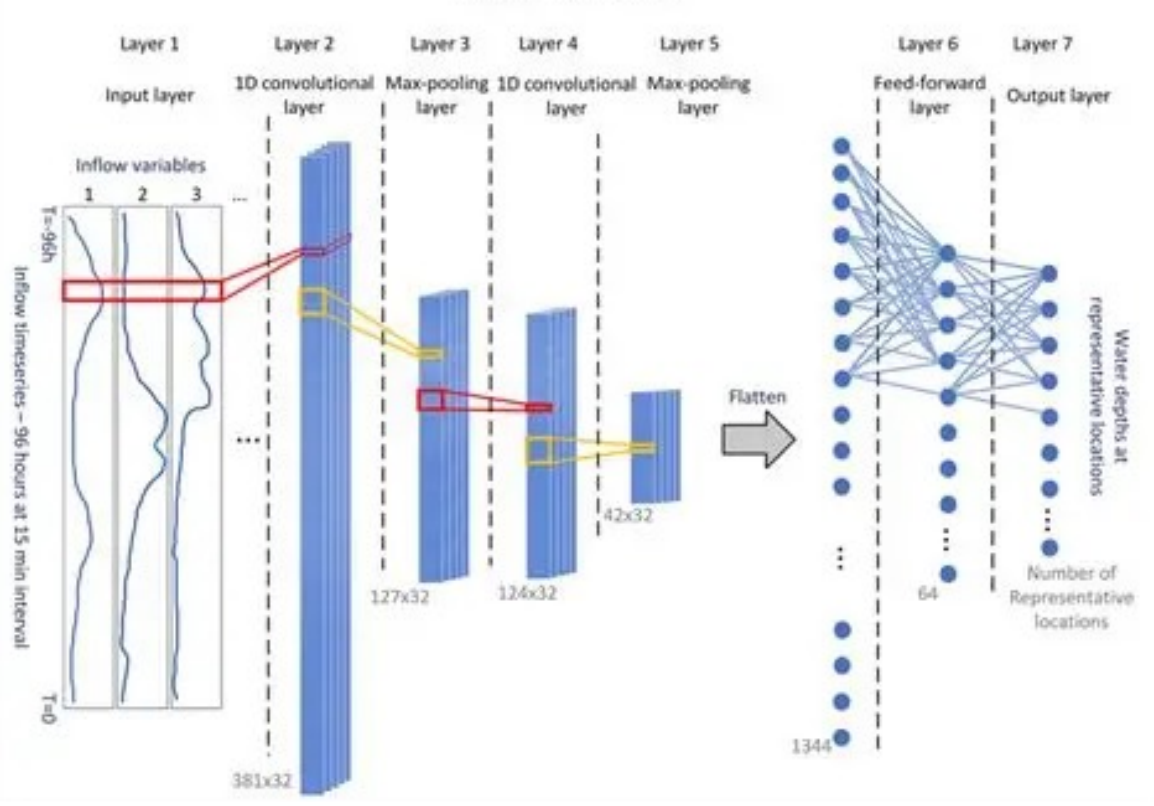
Overall Architecture - 1D Example





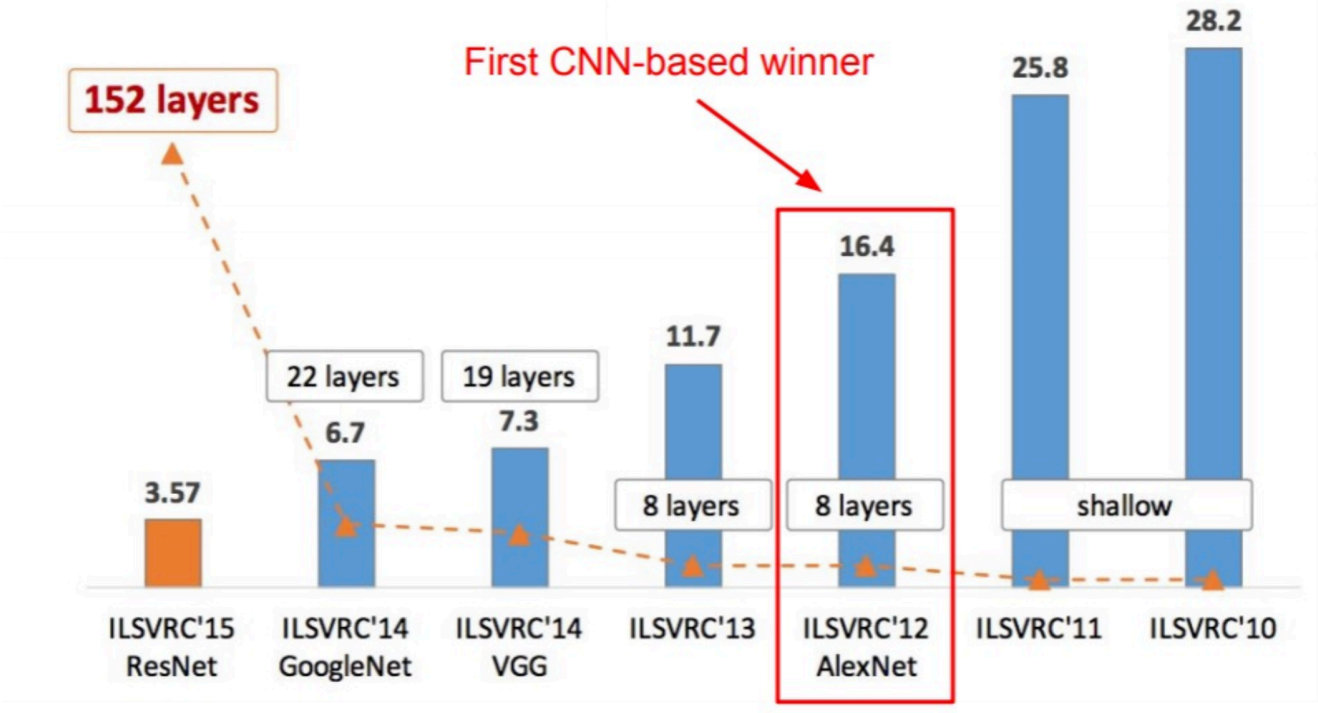
Convolutional Neural Networks

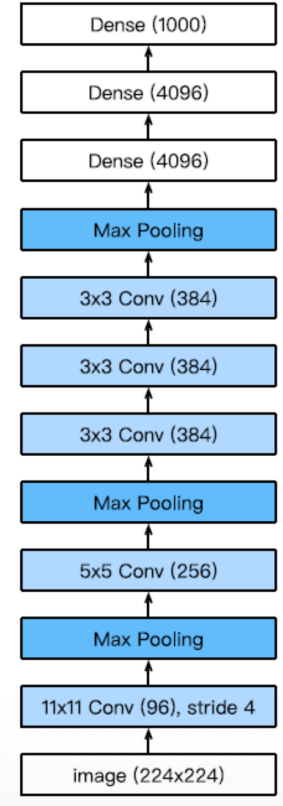
Overall Architecture - 1D Example



Historical Architectures

ImageNet Large Scale Visual Recognition Challenge (ILSVRC) Winners





Historical Architectures

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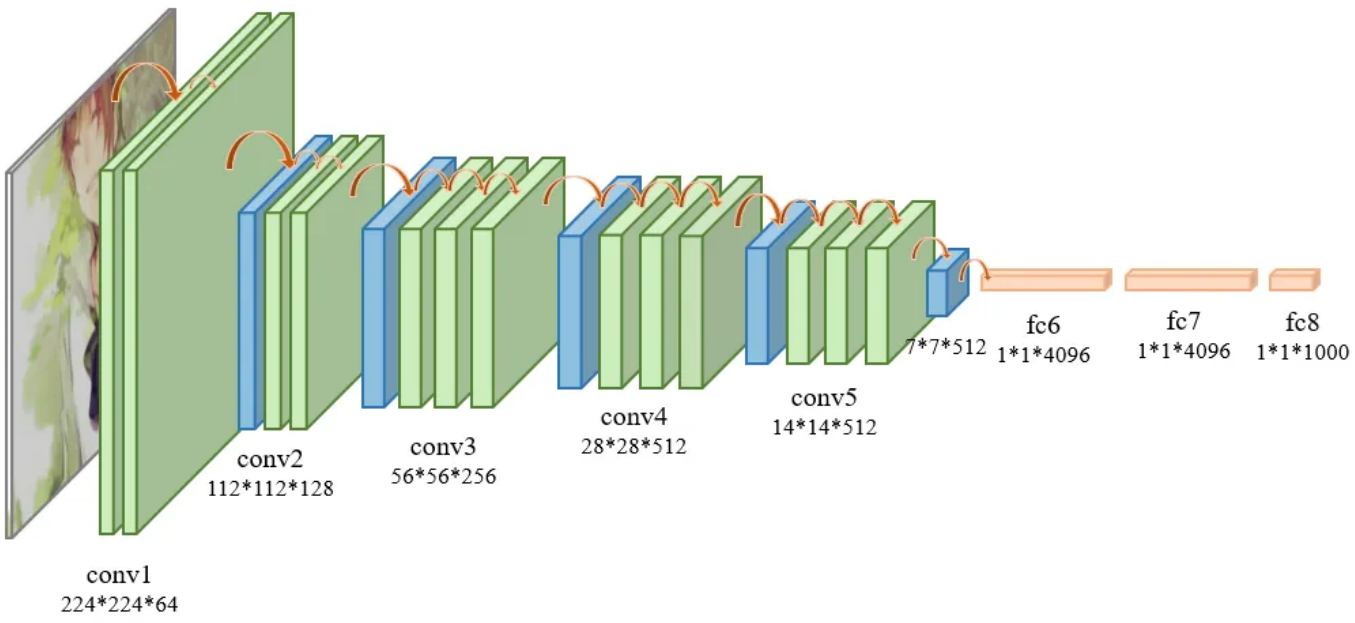


Data Augmentation for Images

Geometric Transforms

- **Random crop:** Crop random regions from larger image
- **Random horizontal flip**: P=0.5 usually
- **Random rotation:** Small angles (±15°)
- Random scale/zoom
- Random affine transforms

Data Augmentation for Images

Color Transforms

- **Color jitter:** Randomly adjust brightness, contrast, saturation, hue
- **Random grayscale:** Sometimes convert to grayscale
- Channel shuffle/drop

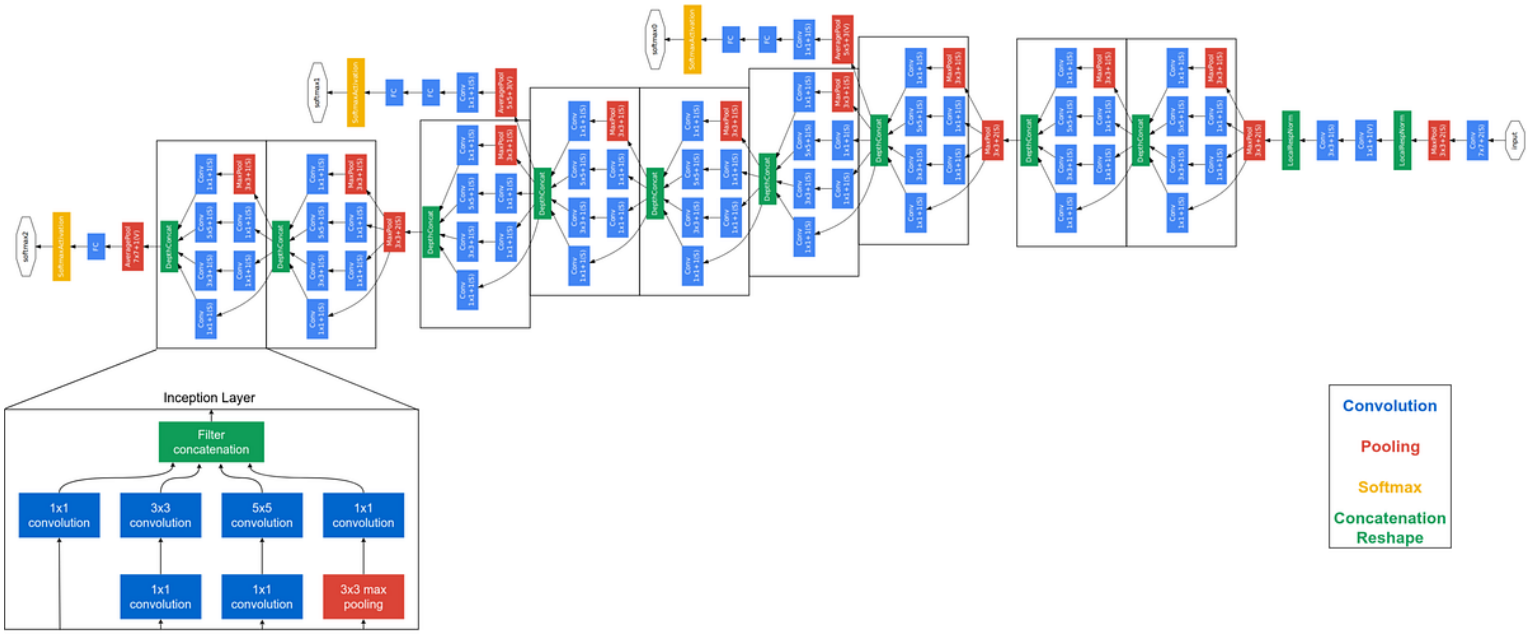
Data Augmentation for Images

Advanced Augmentations

- **Cutout:** Randomly mask square regions
- **Mixup:** Blend two images and their labels
- **CutMix:** Cut and paste patches between images

Transfer Learning

- Training from scratch requires:
  - Millions of labeled images
  - Days of GPU time
  - Expertise in architecture design

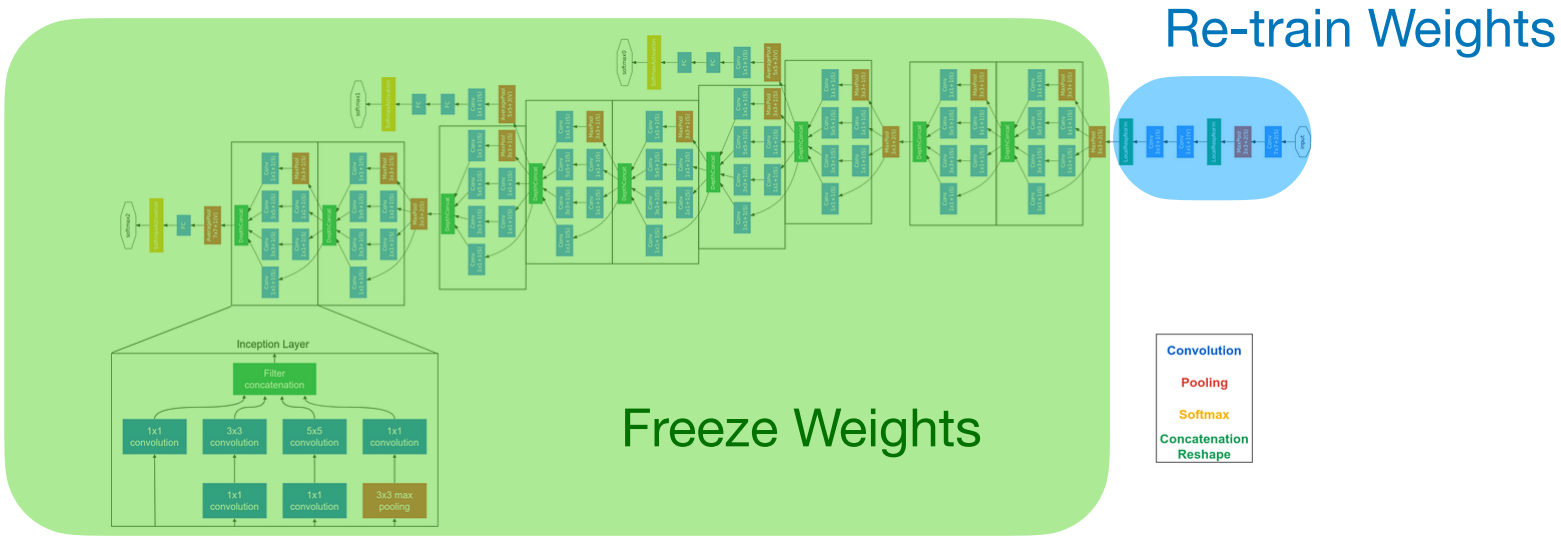


Transfer Learning

- GoogLeNet pre-trained models have learned:
  - Low-level features (edges, textures) - Layer 1-2
  - Mid-level features (shapes, parts) - Layer 3-4
  - High-level features (object parts) - Layer 5+
- These features are **transferable** to many vision tasks

Transfer Learning

- These features are **transferable** to many vision tasks
- Key Idea:
  - Freeze pretrained layers, train only new classifier



Transfer Learning

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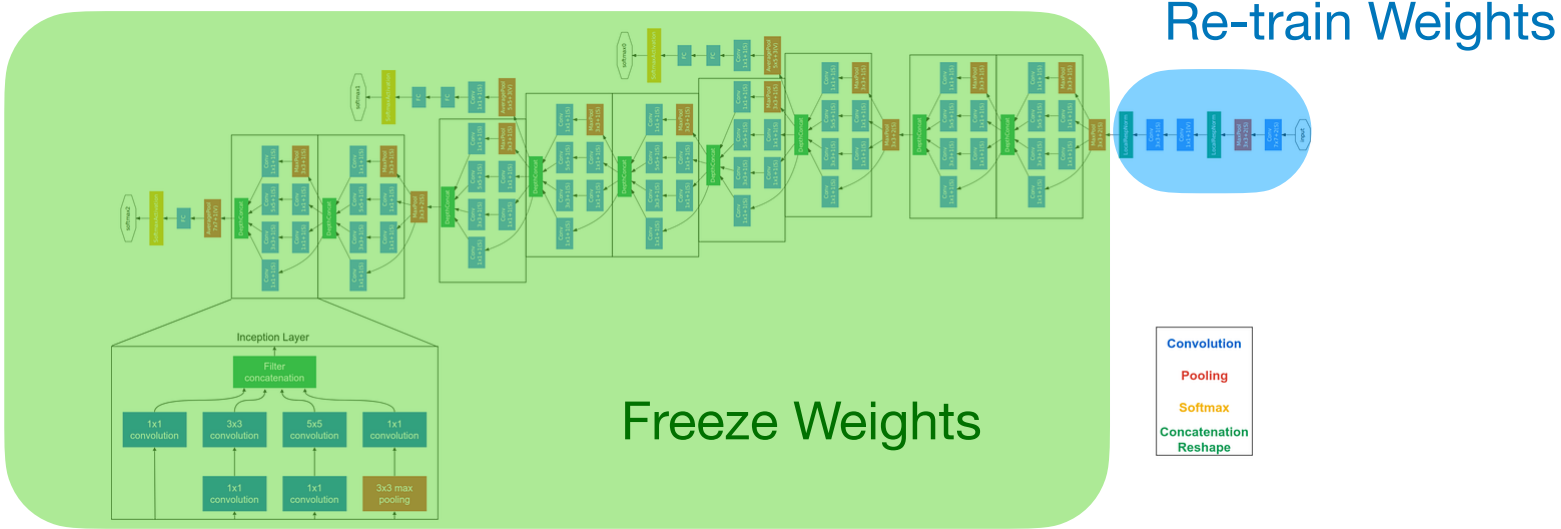
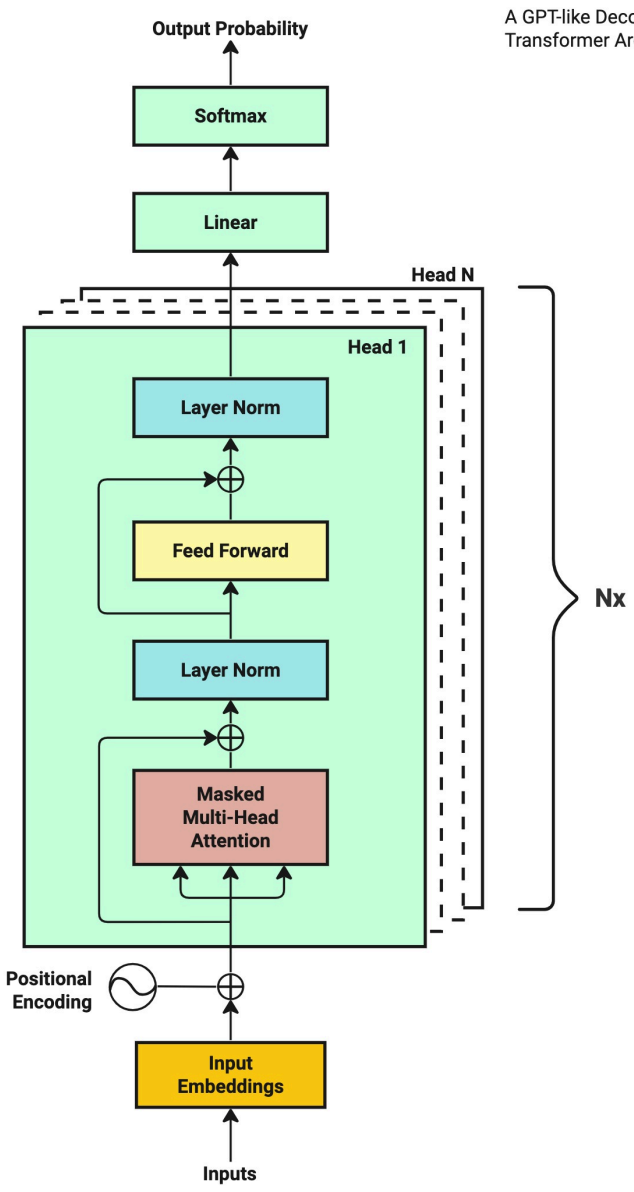
- Key Idea:
  - Freeze pretrained layers, train only new classifier



Transfer Learning

Current LLM Architecture

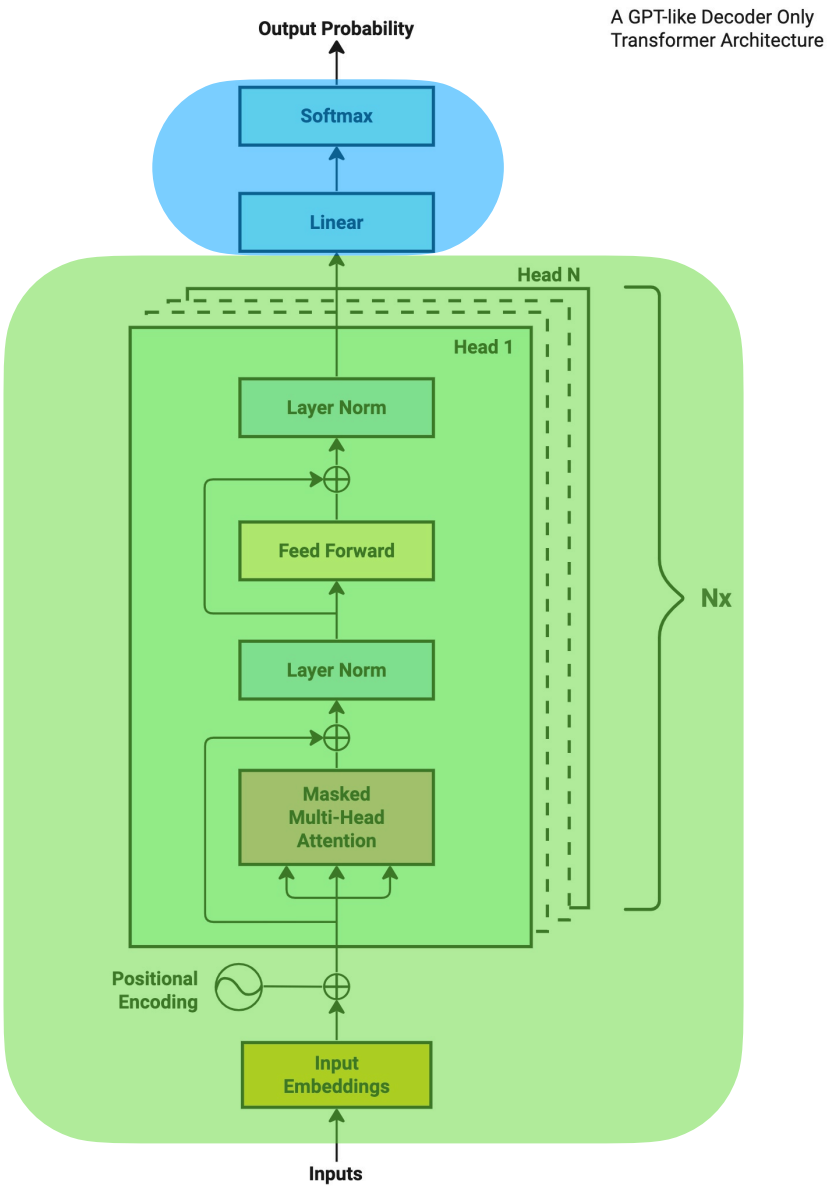


Transfer Learning

Current LLM Architecture

A simple fine-tuning approach:

1. Keep initial layers fixed
2. Retrain final prediction layers



Summary

- Applicable to data with natural grid topology
  - Time series
  - Images
- Convolution is a linear operation that uses local information
- Used for dimensionality reduction and learning hierarchical feature representations
- Much fewer parameters relative to Feed-Forward Neural Networks
- Reached human-level performance in ImageNet in 2014